## Question

The description of a property line says to go: “200 feet to a half-inch iron pin;”

A surveyor finds a half-inch iron pin at 175.24 feet, Figure 1. Does this pin fit the description?

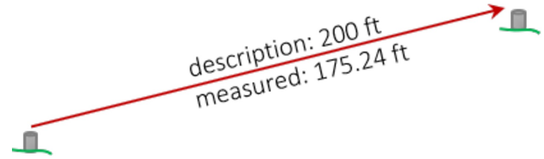



*Figure 1: Record vs Measured*

# I. Numbers

## A. Type

Exact

Inexact

## B. Communication

Magnitude

Accuracy

## C. Significant Figures

### *1. Definition*

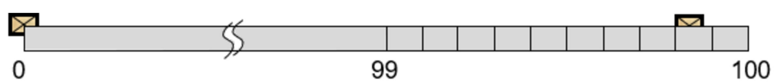



*Figure 2: Reading a Tape*

### *2. Which digits are significant*

### *3. Math operations*

Addition/subtraction:

Multiplication/Division:

***4. Computation errors***

# II. Measurement Errors

## A. Quality

***1. Precision***

***2. Accuracy***

***3. Resolution***

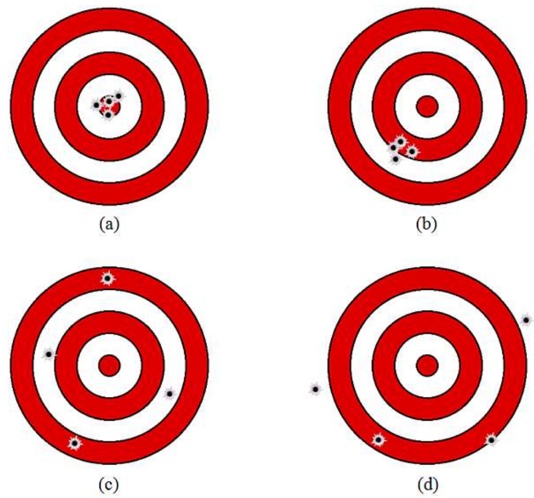



*Figure 3: We Love Shootin' Stuff*

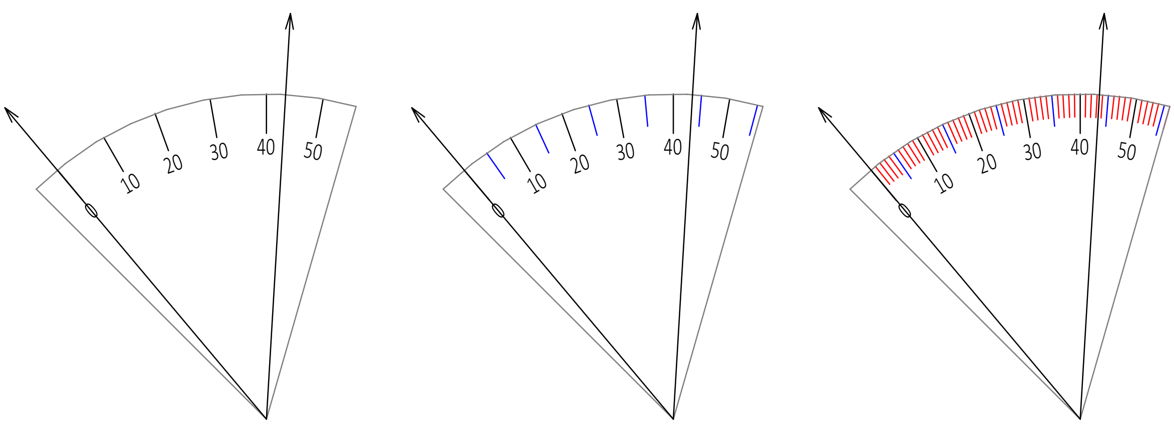



*Figure 4: Measurement Resolution*

## B. Single Measurement Error Tenets

No measurement is exact.
Every measurement contains errors.
The true value of a measurement is never known.
The exact error present in a measurement is unknown.

## C. Error Sources

### *1. Natural*

*Figure 6: Into Each Life Some Rain Must Fall*

*Figure 5: He's Very Friendly...*

### *2. Instrumental*

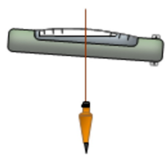


*Figure 7: Bubble Run*

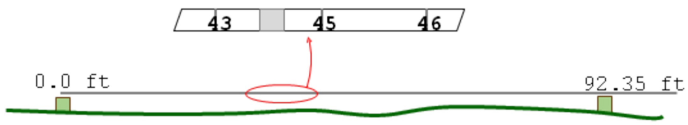



*Figure 8: Mended Tape*

### *3. Personal*

*Figure 9. Unlevel Level*

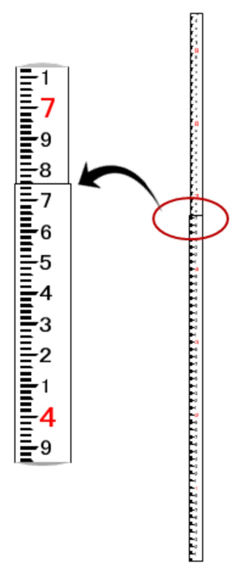



*Figure 10: Not Fully Extended*

## D. Error Types

### *1. Mistake*

***2. Systematic***

***3. Random***

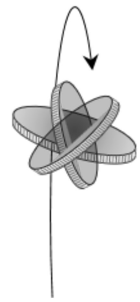


*Figure 11: Coin Flipping*

## E. Behavior

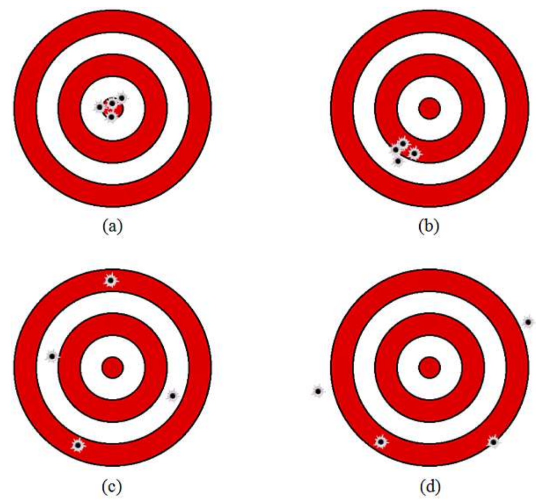



*Figure 12. Errors Affecting **Groupings***

## F. Minimizing

***1. Mistake***

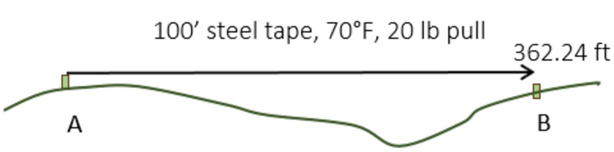



*Figure 13: Distance AB*

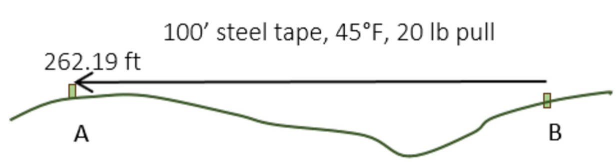



*Figure 14: Distance BA*

Remeasure until within acceptable tolerance.

Don’t use any measurement outside the tolerance.

### *2. Systematic*

#### a. Adjustment

#### b. Computation

#### c. Procedure

We follow specific procedures in surveying to:

- ensure measurement is made correctly
- allow some systematic errors to cancel

#### d. Example

Level collimation error: Caused by cross-hairs set too high or low

Peg-test: used to determine amount of collimation

*Adjustment*

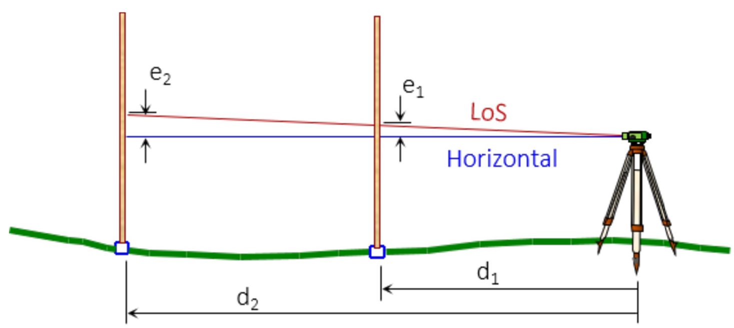
Run peg-test; Raise or lower cross-hair reticule to correct

*Computation*

Run peg-test; Determine collimation error rate, c

For any rod reading, e = d x c; add to reading

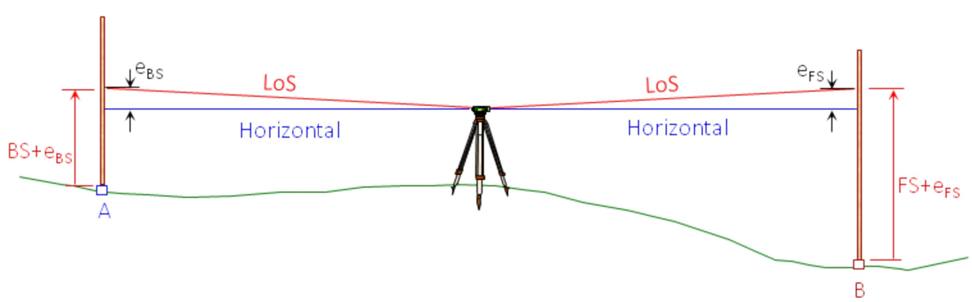
Requires distance to rod




$d_2 = 2(d_1)$
$e_2 = 2(e_1)$

*Figure 15 - Level Collimation Error*

*Procedure*




$$Elev_B = Elev_A + (BS + e_{BS}) - (FS + e_{FS})$$
$$Elev_B = Elev_A + BS + e_{BS} - FS - e_{FS}$$
$$Elev_B = Elev_A + BS - FS + e_{FS} - e_{FS}$$
$$e_{BS} = e_{FS}$$
$$\Rightarrow Elev_B = Elev_A + BS - FS$$

Balance BS and FS distances; error cancels

*Figure 16: Procedural Collimation Compensation*

#### e. Other Procedural Compensations?

*Leveling*

Earth curvature; Refraction, Figure 17.

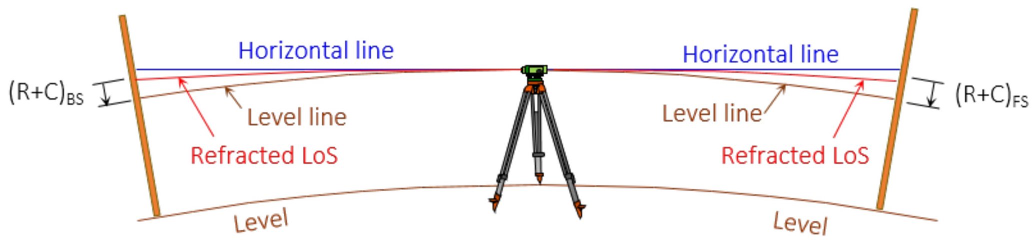



*Figure 17: Balancing BS and FS Distances*

*Total Station*

Plate bubble run
Optical plummet
Double centering
Measuring horizontal angle direct and reverse

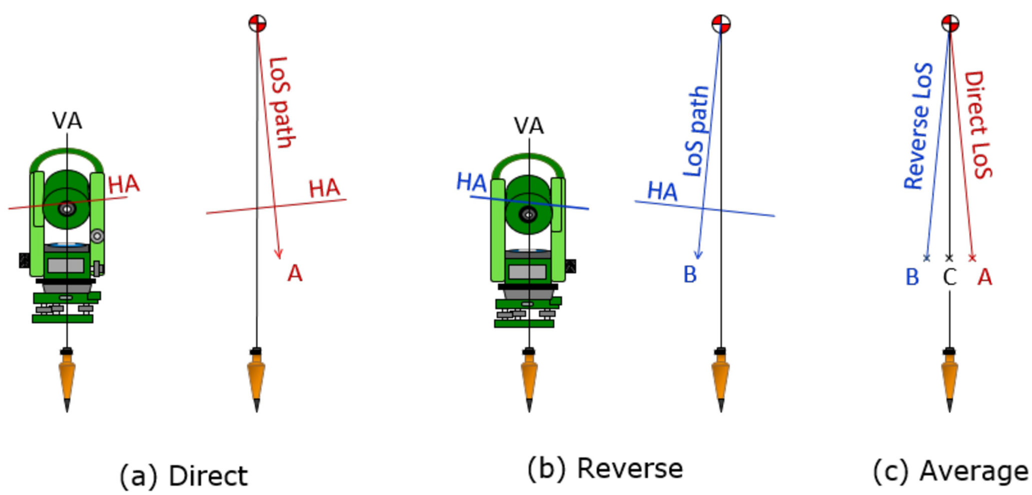



*Figure 18: Principle of Reversion*

### *2. Random*

Repeat measurements sufficient times to allow ± errors to compensate.

Example: FGCS Triangulation horizontal angle measurement specifications.

| Order/Class | 1st | 2nd / I | 2nd / II | 3rd / I | 3rd / II |
|---|---|---|---|---|---|
| Num Positions | 16 | 16 | 8 or 12* | 4 | 2 |
| Max Std Dev of Mean | 0.4” | 0.5” | 0.8” | 1.2” | 2.0” |
| Reject Limit from Mean | 4” | 4” | 5” | 5” | 5” |

*8 if 0.2” resolution instrument, 12 if 1”

*Independent* measurements, to

prevent repeating mistakes

allow systematic error cancellation

Example: $BS_A$ & $FS_B$ then $BS_B$ & $FS_A$ from same instrument set up, Figure 19, are not independent measurements.

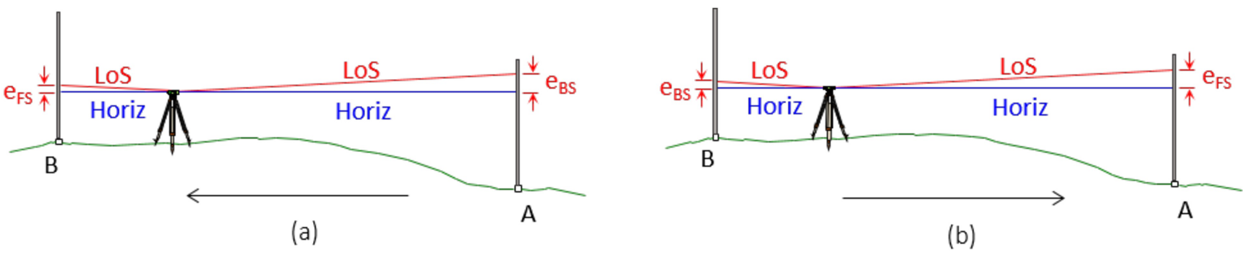



*Figure 19: Non-Independent Measurements*

Repeating measurements: How many times?

Figure 20 compares 3 different resolution instruments. Each D/R set is two angle measurements. Initial repetition greatest benefit; eventual diminishing returns.

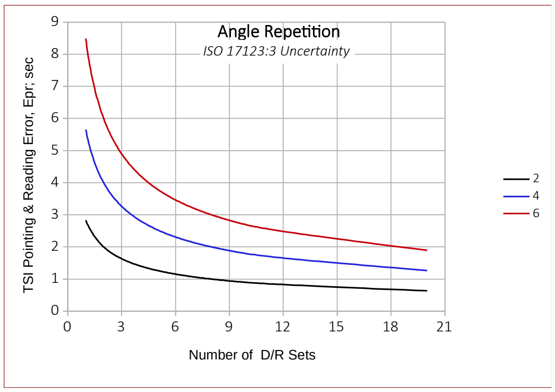



*Figure 20: Angle Repetition*

# III. Random Errors

## A. Basic Analysis

Example Measurement set: 45.66 45.66 45.68 45.65

### *1. Terms*

**Direct and Indirect Measurements**

**Redundancy**

**Discrepancy**

**Most probable value**

$$MPV = \frac{\sum m}{n}$$

**Residual**

$$v = MPV - m$$

**Least Squares**

$$\sum\left(v^2\right) = \min$$

**Standard deviation**

$$\sigma = \sqrt{\frac{\sum\left(v^2\right)}{n-1}}$$

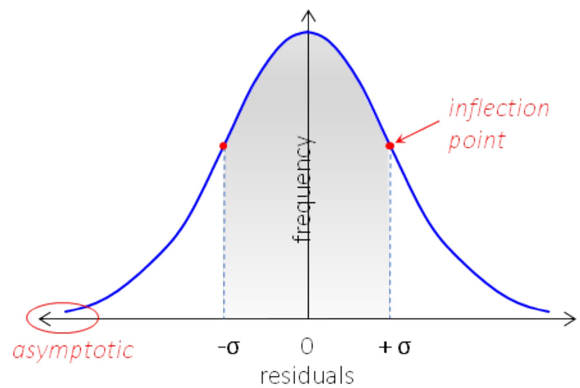



Figure 21: Normal Distribution Curve

**e. Confidence interval**

**f. Standard Error Of the Mean**

$$E_{MPV} = \frac{\sigma}{\sqrt{n}}$$

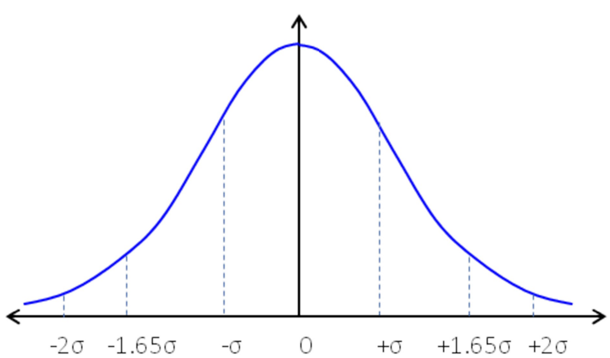



Figure 22: Confidence Intervals

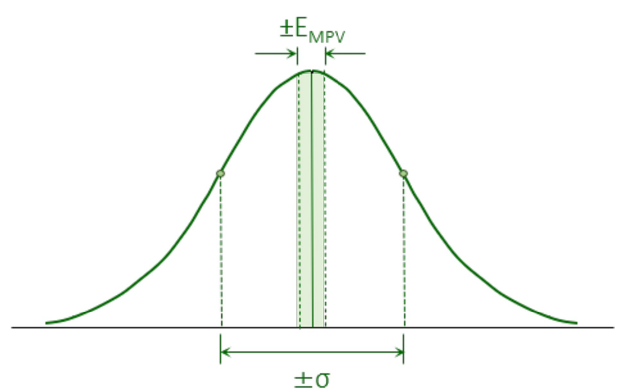



Figure 23: Error of the Mean

### *2. Example*

| num | m | v, residual | $v^2$ |
|---|---|---|---|
| 1 | 45.66 | | |
| 2 | 45.68 | | |
| 3 | 45.66 | | |
| 4 | 45.65 | | |
| sums: | | | |

### *3. Weights*

Mixing different quality measurements.

$$MPV = \frac{\sum (w \times meas)}{\sum w}$$

Relative weight is inversely proportional to standard deviation squared ($w = 1/s^2$). For distances, it can also be inversely proportional to precision ($w = 1/p$).

Example: A distance was independently measured multiple times by three crews. Their results are in the table.

| Crew | Dist (ft) | Std Dev (ft) | Weight | w | w x m |
|---|---|---|---|---|---|
| 1 | 351.11 | ±0.15 | | | |
| 2 | 351.92 | ±0.38 | | | |
| 3 | 352.35 | ±0.08 | | | |
| sums | | | | | |

### *4. Comparisons*

Two survey crews measure different horizontal angles multiple times:

| | Crew A | Crew B |
|---|---|---|
| Num of meas | 2 D/R | 4 D/R |
| Average angle | 128°18’15” | 196°02’40” |
| Std Dev | ±0°00’12” | ±0°00’14” |

Which crew has better precision? Why?

Which crew has better expected accuracy? Why?

### *5. Random Errors Angle Comps*

**a. Example 1**

| num | DMS | DD: 3 dp | DD: 7dp | Sec* |
|---|---|---|---|---|
| 1 | 35°15'30" | 35.258° | 35.2583333° | 30" |
| 2 | 35°15'56" | 35.266° | 35.2655556° | 56" |
| 3 | 35°15'35" | 35.260° | 35.2597222° | 35" |
| 4 | 35°15'47" | 35.263° | 35.2630556° | 47" |
| | sums | 141.047° | 141.0466667° | 168" |
| | MPV | 35.262° | 35.2616667° | 42.0" |
| | | 35°15'43.2" | 35°15'42.0" | +35°15' |
| | | | | 35°15'42.0" |
| | Std Dev | ±0.004° | ±0.0032632° | |
| | | ±14.4" | ±11.7" | ±11.7" |

* 35°15' is constant for each angle, so subtract it and work with just the seconds.

**b. Example 2**

| num | DMS | Sec | v | $v^2$ |
|---|---|---|---|---|
| 1 | 107°03'58" | | | |
| 2 | 107°04'01" | | | |
| 3 | 107°04'03" | | | |
| 4 | 107°03'59" | | | |
| | sums | | | |

## B. Error Propagation

### *1. Simple*

**a. Error of a Sum**

Adding or subtracting values which are subject to errors.

$$E_{sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

*Example:*

| Soil sample | Weight |
|---|---|
| 1 | 56.2 gr ±0.15 gr |
| 2 | 28.9 gr ±0.21 gr |
| 3 | 41.6 gr ±0.11 gr |

Total weight = 126.7 gr ± ______ gr

**b. Error of a Series**

The same error occurring multiple times.

$$E_{series} = E\sqrt{n}$$

*Example:* Each time they set up a level, a survey crew can determine an elevation difference to ±0.015 ft. What is their expected misclosure error on this loop?

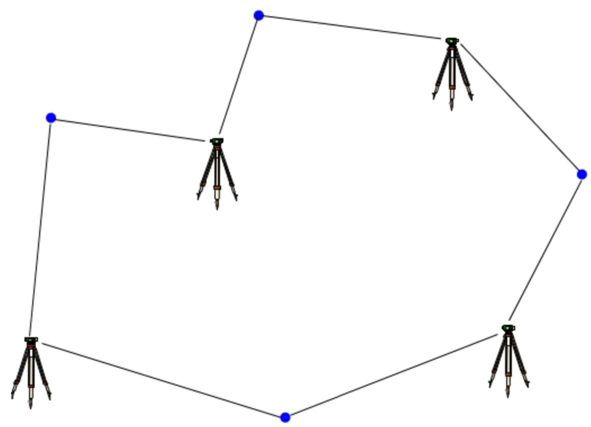


*Figure 24: Level Loop Example*

**c. Error of a Product:**

Two values with errors are multiplied or divided.

$A \pm E_A$, $B \pm E_B$

$$E_{prod} = \sqrt{(A \times E_B)^2 + (B \times E_A)^2}$$

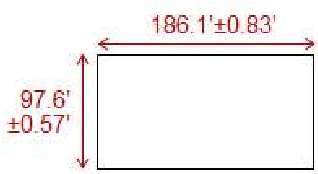



*Figure 25. Area Error*

### *2. Combined*

#### a. Distance

$$E_{dist} = \sqrt{E_{si}^2 + C_{msa}^2 + \left(\frac{P_{msa} x D}{1,000,000}\right)^2 + E_{ref}^2}$$

TSI setup error: ±0.005 ft; handheld reflector centering: ±0.01 ft; 712.36 ft measured distance.

MSA: ±(3 mm + 3 ppm).

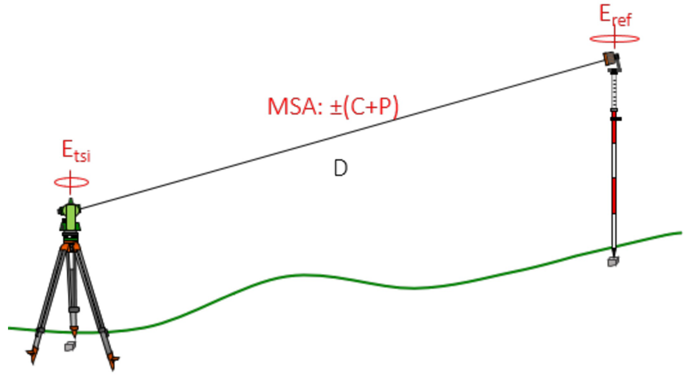



*Figure 26. Distance Error*

#### b. Horizontal TSI Angle

$$E_{pr} = \frac{2 \times E_{DIN}}{\sqrt{n}}$$

$$E_{tsi} = \frac{D \times E_i}{D_{BS} D_{FS} \sqrt{2}} \times \frac{206,264.8 \text{ sec}}{\text{radian}}$$

$$E_t = \sqrt{\left(\frac{E_{BS}}{D_{BS}}\right)^2 + \left(\frac{E_{FS}}{D_{FS}}\right)^2} \times \frac{206,264.8 \text{ sec}}{\text{radian}}$$

$$E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$$

Angle: 123°30’10” meas’d 2 D/R; TSI DIN: 2 sec; TSI centering error: ±0.005 ft; BS & FS centering errors: ±0.005 ft & ±0.01 ft; BS & FS dists: 176 ft & 243 ft.

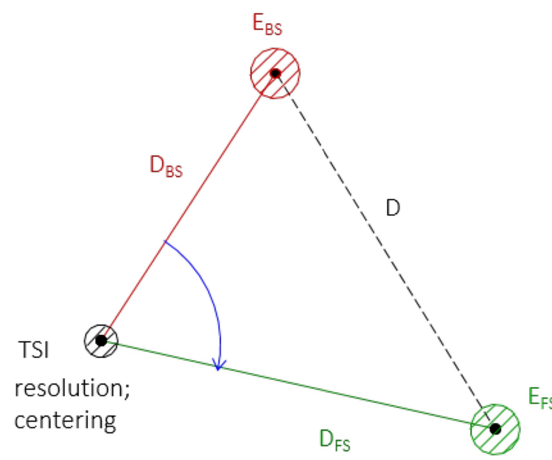



*Figure 27. Horizontal Angle Error*

# IV. Least Squares Adjustment (LSA)

## A. Compared to Simplified Adjustments

### *1. Advantages*

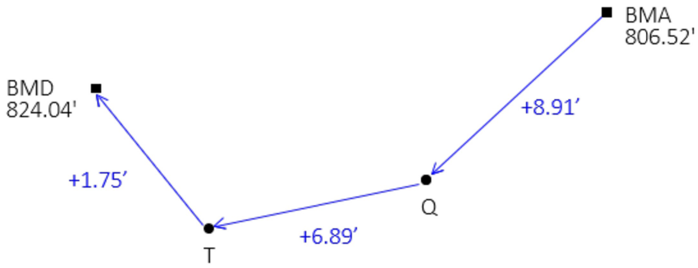
Better modeling of random error behavior
Able to deal with multiple unknowns simultaneously
Easily incorporates redundant measurements - the more the merrier
Can use mixed quality measurements
Can generate statistics for overall adjustment and individual solved unknowns

### *2. Disadvantages*

Computation intensive
Statistics overload
Easy to misuse

## B. Building Complex Networks

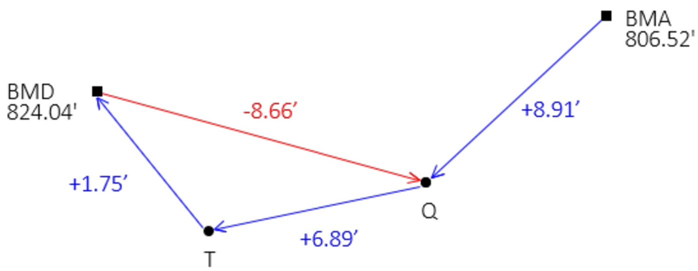
Multiple unknowns




Can apply simple network adjustment.

*Figure 28: Three observations*

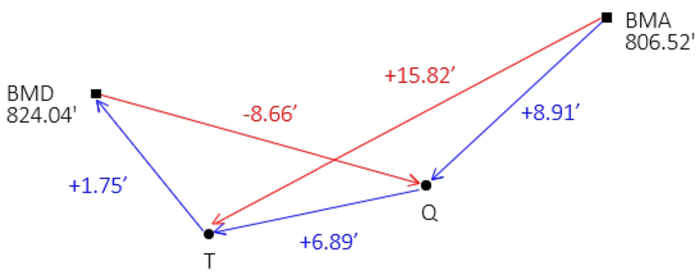
Add fourth observation.




Simple network adjustment is cumbersome.

*Figure 29: Fourth Observation Added*

Add fifth observation.




Simple network adjustment not possible.

*Figure 30: Fifth Observation Added*

## C. Random Errors Only

Mistakes and systematic errors must be eliminated/compensated.

Mistake - effect can be spread into other measurements, Figure 31

Systematic errors - residual sizes and distribution depend on:

Error character - scale or additconstant

Which measurements are affected

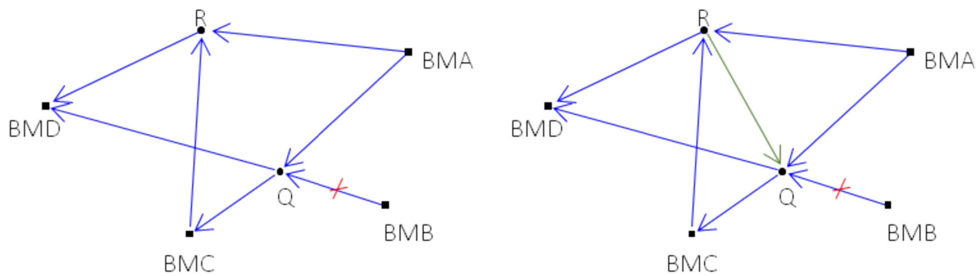



*Figure 31: Mistake Inclusion*

## D. Redundancies

### *1. General*

Redundancy aka degree of freedom (df): df = m-n

m: number of measurements
n: number of unknowns

| If... | Then... |
|---|---|
| m<n | |
| m=n | |
| m>n | |

### *2. Vertical Network - 1D*

How many observations are necessary to determine the elevations of points A and B from the benchmark, BM Dog, Figure 32?

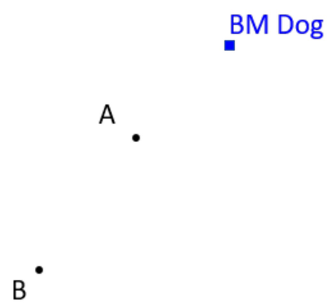



*Figure 32: Elevation Points*

How many df in

Figure 33?
Figure 34?
Figure 35?

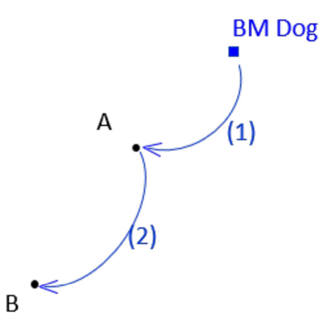



*Figure 33: 2 Observations*

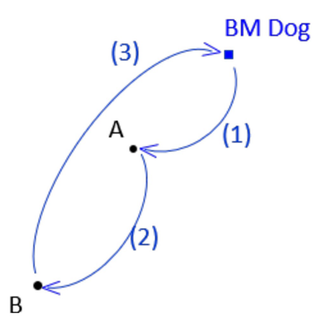



*Figure 34: 3 Observations*

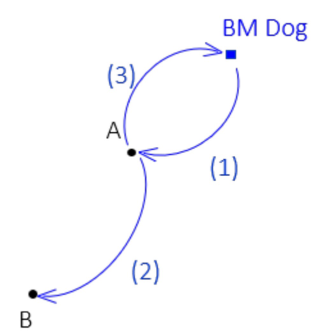



*Figure 35: 3 Observations*

Difference between Figures 34and 35?

Figure 34 - true redundancies
Figure 35- pseudo-redundancies

### *3. Horizontal Network - 2D*

Unknowns are point positions: coordinates.

How many unknowns in Figure 36?

Traditional traverse, Figure 37:

Fix one point and one direction.
Measure sides and angles between
Num df?

Can apply simple network adjustment (eg, Compass Rule, Transit Rule).

Add redundancies?

Fix one point and one direction; Measure sides and angles between.

Num df in
- Figure 38?
- Figure 39?
- Figure 40?

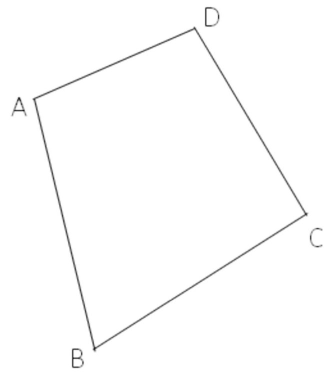



*Figure 36: Lot Corners*

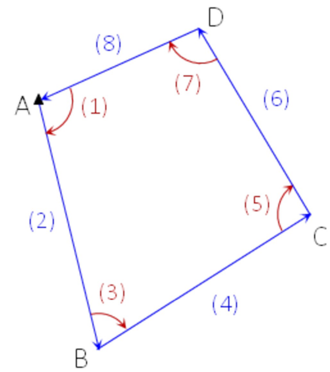



*Figure 37: Traverse*

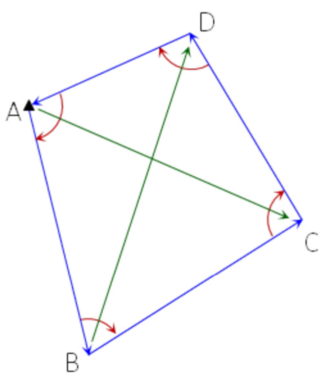



*Figure 38: Add Diagonals*

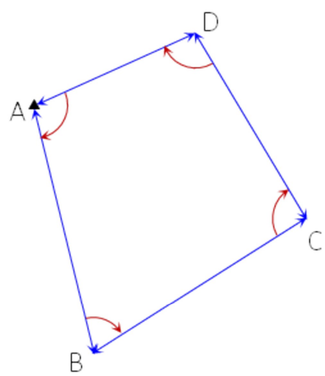



*Figure 39: Distances in Both Directions; Angles*

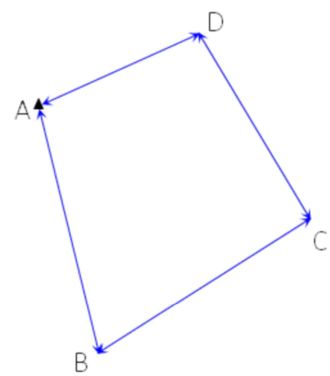



*Figure 40: Distances Only*

True vs pseudo-redundancies

## D. Weights

Sometimes science, sometimes art

### *1. Vertical - Leveling*

Distances
Number of setups

### *2. Horizontal*

**Distances**

$$E_{dist} = \sqrt{E_s^2 + C_{msa}^2 + \left(\frac{P_{msa} x D}{1,000,000}\right)^2 + E_{ref}^2}$$

**Directions and Angles**

$$E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$$

## E. Linear/Non-Linear

An LSA minimizes the sum of the squares of the residuals of the observations: $\Sigma(v^2)$=min.

### *1. Vertical - Linear*

Measurements are simple addition & subtraction: $Elev_B = Elev_A + BS_A - FS_B$

Non-iterative solution.

Example: For each line, write an observation equation which includes a residual term.

$$\begin{aligned} Elev_Q &= 806.52 + 8.91 + v_{aq} \\ Elev_T &= Elev_Q + 6.89 + v_{qt} \\ 824.04 &= Elev_T + 1.75 + v_{tb} \\ Elev_Q &= 824.04 - 8.66 + v_{bq} \end{aligned} \Rightarrow$$

$$\begin{aligned} v_{aq} &= Elev_Q - 815.43 \\ v_{qt} &= Elev_T - Elev_Q - 6.89 \\ v_{tb} &= 822.29 - Elev_T \\ v_{bq} &= Elev_Q - 815.38 \end{aligned}$$

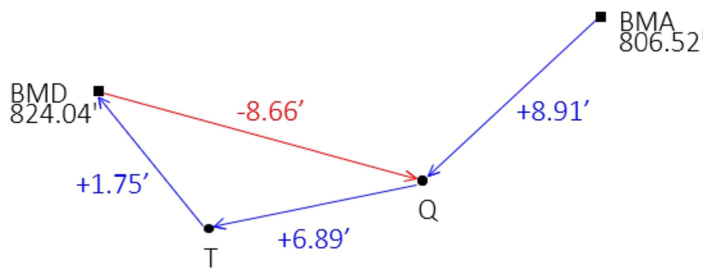



*Figure 41: Level Network Adjustment*

Square the residuals and add them.

$$F = \sum_1^n v_i^2 = (Elev_Q - 815.43)^2 + (Elev_T - Elev_Q - 6.89)^2 + (822.29 - Elev_T)^2 + (Elev_Q - 815.38)^2$$

To minimize residuals, the partial derivative of the function with respect to each unknown elevation is set equal to 0.

$$\frac{\partial F}{\partial Elev_Q} = 2(1)(Elev_Q - 815.43) + 2(-1)(Elev_T - Elev_Q - 6.89) + 2(1)(Elev_Q - 815.38) = 0.000$$

$$\cdots \Rightarrow 3(Elev_Q) - Elev_T - 1623.92 = 0.000$$

$$\frac{\partial F}{\partial Elev_T} = 2(1)(Elev_T - Elev_Q - 6.89) + 2(-1)(822.29 - Elev_T) = 0.000$$

$$\cdots \Rightarrow 2(Elev_T) - Elev_Q - 829.18 = 0.000$$

The two equations can be solved simultaneously for the unknown elevations.

$Elev_Q$ = 815.404', $Elev_T$ = 822.292'

### *2. Horizontal - Non-Linear*

Measurements are angles and distances.

Position determination requires trig which is non-linear.

LSA solution uses coordinate variation

Start with approximate coordinates

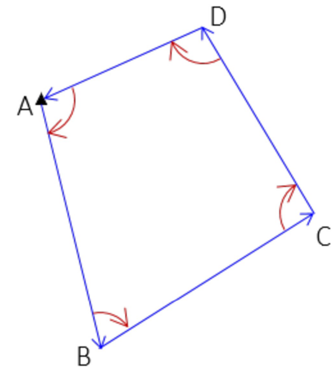



*Figure 42: Horizontal*

Compute and apply corrections
Repeat until corrections are below a threshold

**a. Measurements**

(1) Distances

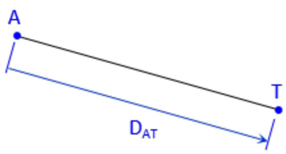



*Figure 43: Distance Measurement*

The residual is the difference between the measured distance and the distance computed from coordinates.

$$v_{at} = D_{AT} - \left[ \left(N_T - N_A\right)^2 + \left(E_T - E_A\right)^2 \right]^{1/2}$$

Four partial derivatives (one for each coordinate) of the residual squared:

$$\frac{\partial D_{AT}}{\partial N_A} = \left[ \frac{-\left(N_T - N_A\right)}{D_{AT}} \right] dN_A \qquad \frac{\partial D_{AT}}{\partial E_A} = \left[ \frac{-\left(E_T - E_A\right)}{D_{AT}} \right] dE_A$$

$$\frac{\partial D_{AT}}{\partial N_T} = \left[ \frac{\left(N_T - N_A\right)}{D_{AT}} \right] dN_T \qquad \frac{\partial D_{AT}}{\partial E_T} = \left[ \frac{\left(E_T - E_A\right)}{D_{AT}} \right] dE_T$$

$dN_A$, $dE_A$, $dN_T$, $dE_T$ are corrections for the initial coordinates.

(2) Angles and Directions

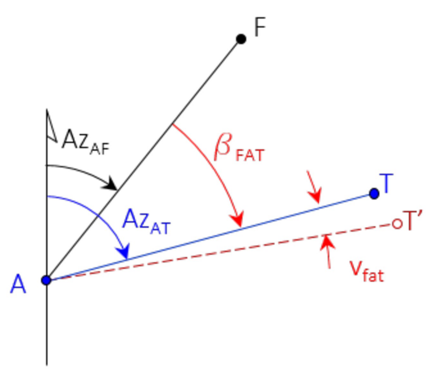



*Figure 44: Angle Measurement*

The residual is the difference between the measured angle and the angle computed from coordinates.

$$v_{fat} = \beta_{FAT} - \left(Az_{AT} - Az_{AF}\right) = \beta_{FAT} - \left( \tan^{-1}\left[ \frac{E_T - E_A}{N_T - N_A} \right] - \tan^{-1}\left[ \frac{E_F - E_A}{N_F - N_A} \right] \right)$$

Six partial derivatives (one for each coordinate) of the residual squared:

$$\frac{\partial \beta_{FAT}}{\partial E_N} = \left[ \frac{\left(E_T - E_A\right)}{D_{AT}^2} - \frac{\left(E_F - E_A\right)}{D_{AF}^2} \right]_O dN_A \qquad \frac{\partial \beta_{FAT}}{\partial E_A} = \left[ \frac{-\left(N_T - N_A\right)}{D_{AT}^2} - \frac{-\left(N_F - N_A\right)}{D_{AF}^2} \right] dE_A$$

$$\frac{\partial \beta_{FAT}}{\partial N_F} = \left[ \frac{\left(E_F - E_A\right)}{D_{AF}^2} \right] dN_F \qquad \frac{\partial \beta_{FAT}}{\partial E_F} = \left[ \frac{-\left(N_F - N_A\right)}{D_{AF}^2} \right] dE_F$$

$$\frac{\partial \beta_{FAT}}{\partial N_T} = \left[ \frac{-\left(E_T - E_A\right)}{D_{AT}^2} \right] dN_T \qquad \frac{\partial \beta_{FAT}}{\partial E_T} = \left[ \frac{\left(N_T - N_A\right)}{D_{AT}^2} \right] dE_T$$

$dN_A$, $dE_A$, $dN_F$, $dE_F$, $dN_T$, $dE_T$ are corrections for the initial coordinates.

**b. Iterative**

Requires initial coordinate approximations

Corrections are combined at points common to angles and distance measurements.

Iterates correcting coordinates until acceptable level is reached.

Needless to say, this can be quite a lot of calculations which is why we use software.

## F. Adjustment Statistics

### *1. Overall*

Standard Deviation of Unit Weight ($S_O$): $S_o = \sqrt{\dfrac{\sum_{i=1}^{m}\left(v_i^2\right)}{(m-n)}}$

### *2. Unknowns*

**a. Vertical**

$S_{Elev}$

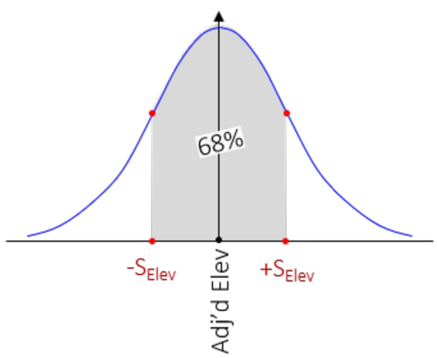



*Figure 45: Adjusted Elevation*

**b. Horizontal**

$S_N$ & $S_E$

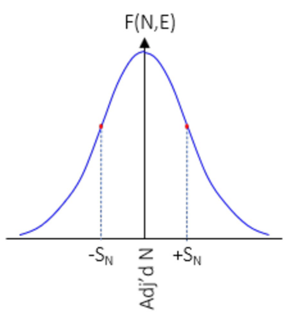



*Figure 46: Adjusted North Coordinate*

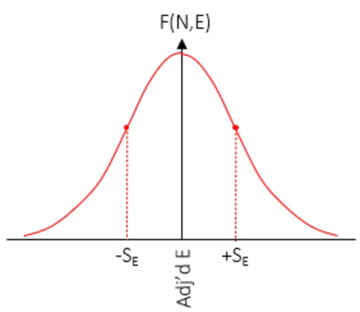



*Figure 47: Adjusted East Coordinate*

Bi-variate; Standard Error Ellipse

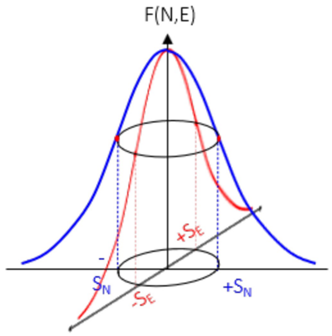



*Figure 48: Bi-Variate Distribution*

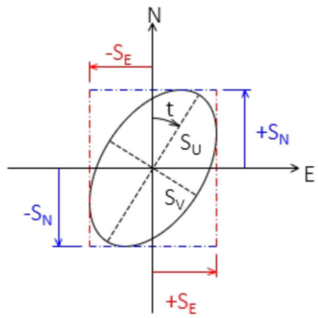



*Figure 49: Standard Error Ellipse*

Standard error ellipse is ~33-35% CI

Confidence Intervals

CI = f(df): $S_{i@CI\%} = S_i \times \sqrt{2 \times F}$

@95% CI:

| DF | F | DF | F |
|---|---|---|---|
| 1 | 199.5 | 4 | 6.94 |
| 2 | 19.0 | 5 | 5.79 |
| 3 | 9.55 | 10 | 4.10 |

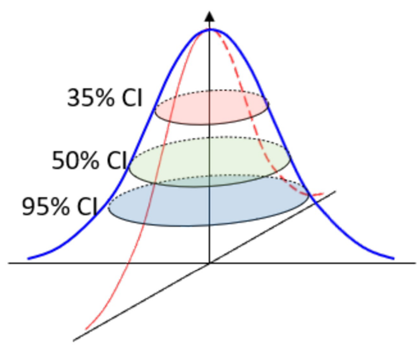



*Figure 50: Different CI*

**c. Adjustment Considerations**

(1) Adjacent Adjustments

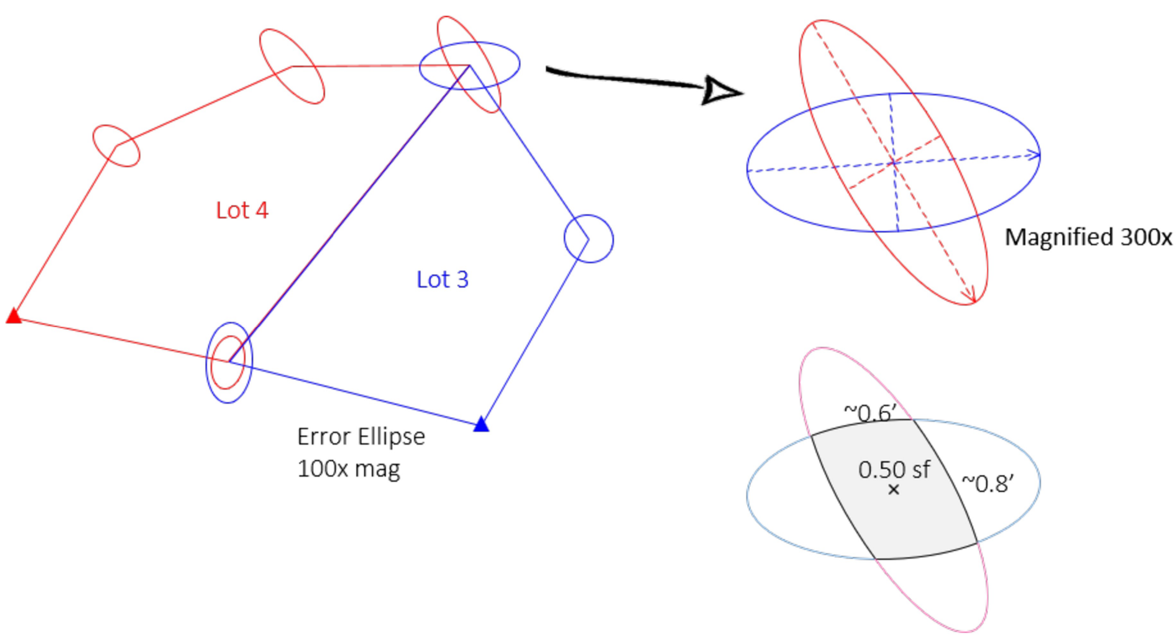



*Figure 51: Overlapping Error Ellipses*

Pin-cushions (monument nests), Figure 52?


*Figure 52: Pin-Cushions*

(2) Noise level?

Are some adjusted values significant or not?

Commencing at a 2" iron pipe at the west quarter corner of Section 31, T5N, R10E;
Thence S88°16'52"W, 0.30 feet to the existing east line of Section 1, T5N, R9E;
Thence S00°18'01"W, 0.01 feet along said east line of said Section 1;
Thence S00°18'01"W, 33.20 feet along said east line;
Thence N88°34'15"E, 33.78 feet to the existing east right-of-way line of STH 104, also being the point of beginning;
Thence N88°47'53"E, 803.03 feet along the existing south right-of-way line of STH 92;
Thence N88°17'20"E, 55.46 feet along said south right-of-way line;

*Figure 53: Statistically Significant?*