Question

The description of a property line says to go: "200 feet to a half-inch iron pin;"

A surveyor finds a half-inch iron pin at 175.24 feet, Figure 1. Does this pin fit the description?

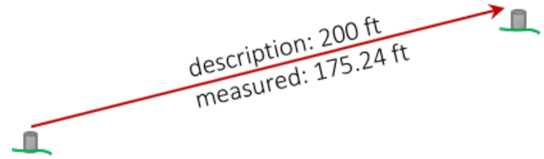


Figure 1: Record vs Measured

I. Numbers

A. Type

Exact

Inexact

B. Communication

Magnitude

Accuracy

C. Significant Figures

1. Definition

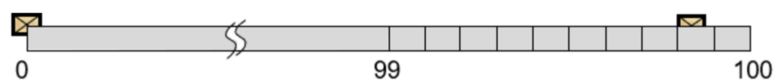


Figure 2: Reading a Tape

2. Which digits are significant

3. Math operations

Addition/subtraction:

Multiplication/Division:

4. Computation errors

II. Measurement Errors

A. Quality

1. Precision

2. Accuracy

3. Resolution

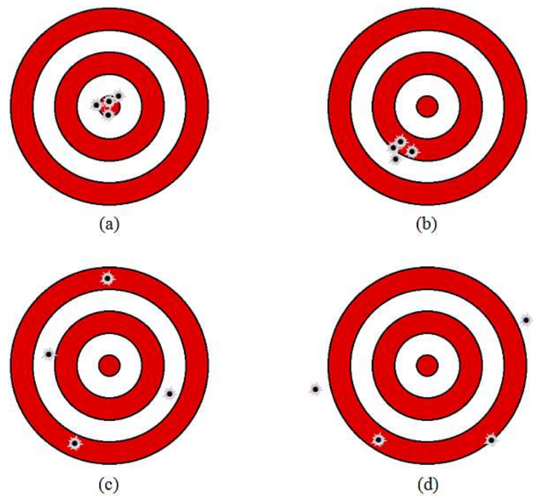


Figure 3: We Love Shootin' Stuff

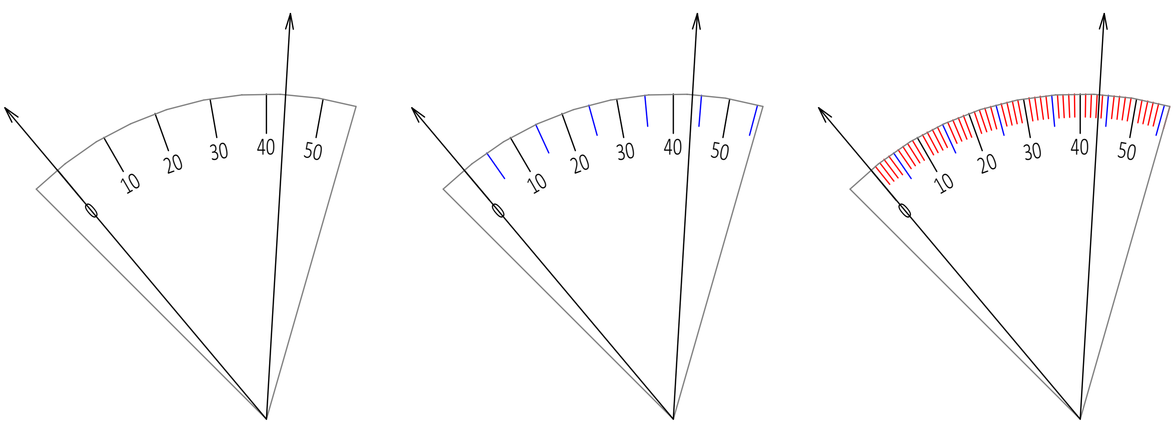


Figure 4: Measurement Resolution

B. Single Measurement Error Tenets

No measurement is exact.
Every measurement contains errors.
The true value of a measurement is never known.
The exact error present in a measurement is unknown.

C. Error Sources

1. Natural

Figure 5: He's Very Friendly...

Figure 6: Into Each Life Some Rain Must Fall

2. Instrumental

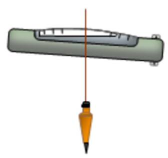


Figure 7: Bubble Run

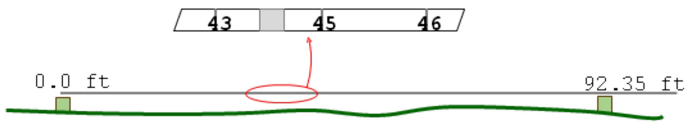


Figure 8: Mended Tape

3. Personal

Figure 9. Unlevel Level

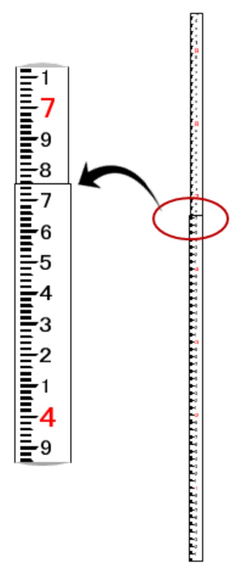


Figure 10: Not Fully Extended

D. Error Types

1. Mistake

2. Systematic

3. Random

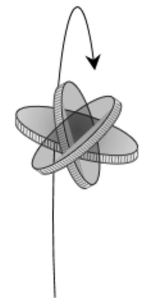


Figure 11: Coin Flipping

E. Behavior

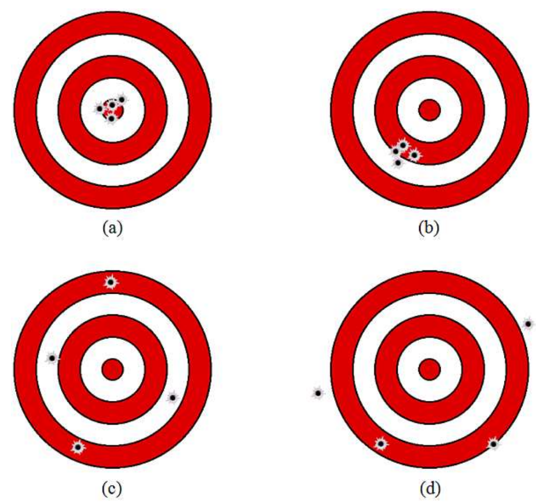


Figure 12. Errors Affecting ***Groupings***

F. Minimizing

1. Mistake

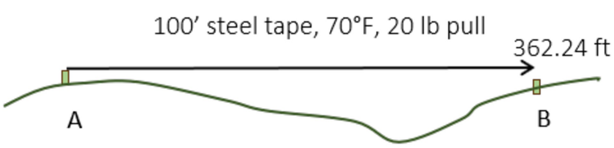


Figure 13: Distance AB

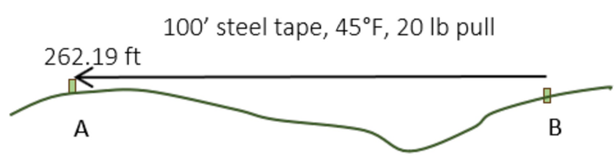


Figure 14: Distance BA

Remeasure until within acceptable tolerance.

Don’t use any measurement outside the tolerance.

2. Systematic

a. Adjustment

b. Computation

c. Procedure

We follow specific procedures in surveying to:

- ensure measurement is made correctly
- allow some systematic errors to cancel

d. Example

Level collimation error: Caused by cross-hairs set too high or low

Peg-test: used to determine amount of collimation

Adjustment

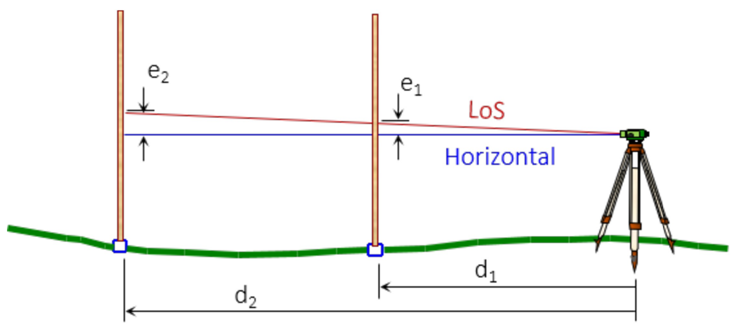
Run peg-test; Raise or lower cross-hair reticule to correct

Computation

Run peg-test; Determine collimation error rate, c

For any rod reading, e = d x c; add to reading

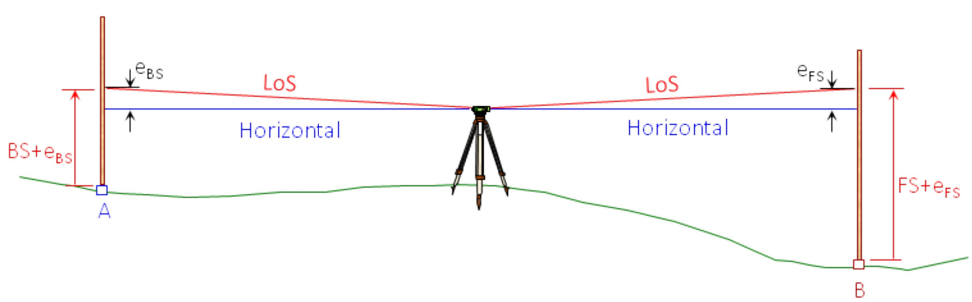
Requires distance to rod



$d_2 = 2(d_1)$
$e_2 = 2(e_1)$

Figure 15 - Level Collimation Error

Procedure



$$Elev_B = Elev_A + (BS + e_{BS}) - (FS + e_{FS})$$
$$Elev_B = Elev_A + BS + e_{BS} - FS - e_{FS}$$
$$Elev_B = Elev_A + BS - FS + e_{FS} - e_{FS}$$
$$e_{BS} = e_{FS}$$
$$\Rightarrow Elev_B = Elev_A + BS - FS$$

Balance BS and FS distances; error cancels

Figure 16: Procedural Collimation Compensation

e. Other Procedural Compensations?

Leveling

Earth curvature; Refraction, Figure 17.

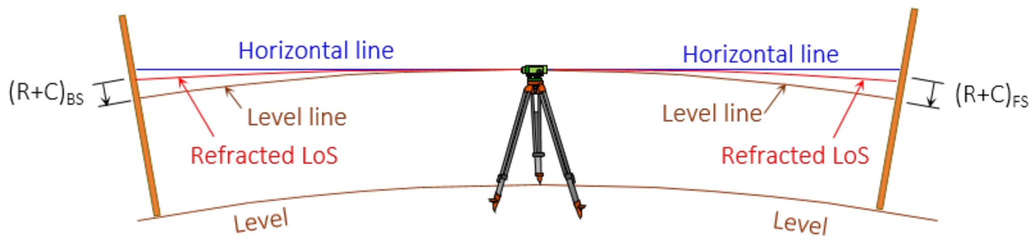


Figure 17: Balancing BS and FS Distances

Total Station

Plate bubble run
Optical plummet
Double centering
Measuring horizontal angle direct and reverse

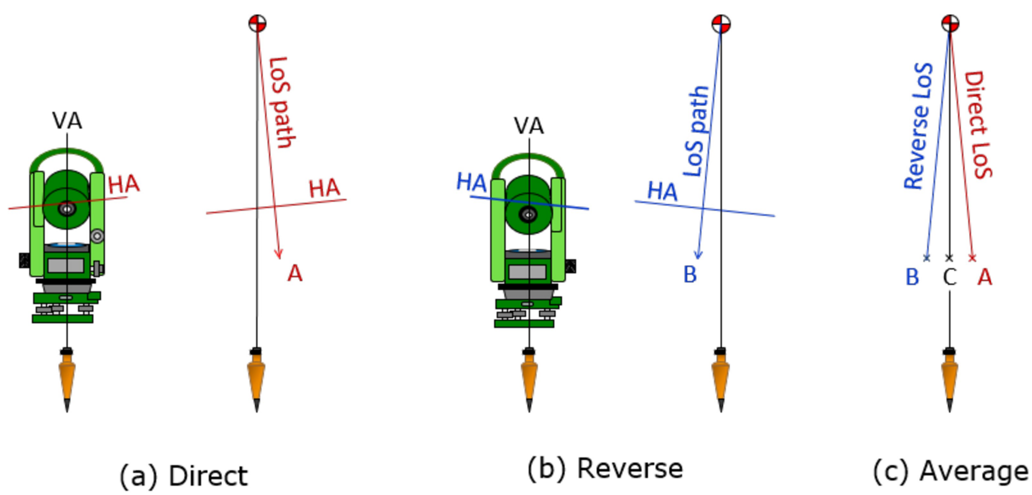


Figure 18: Principle of Reversion

3. Random

Repeat measurements sufficient times to allow ± errors to compensate.

Example: FGCS Triangulation horizontal angle measurement specifications.

Order/Class	1st	2nd / I	2nd / II	3rd / I	3rd / II
Num Positions	16	16	8 or 12*	4	2
Max Std Dev of Mean	0.4”	0.5”	0.8”	1.2”	2.0”
Reject Limit from Mean	4”	4”	5”	5”	5”

*8 if 0.2” resolution instrument, 12 if 1”

Independent measurements, to

prevent repeating mistakes

allow systematic error cancellation

Example: BS_A & FS_B then BS_B & FS_A from same instrument set up, Figure 19, are not independent measurements.

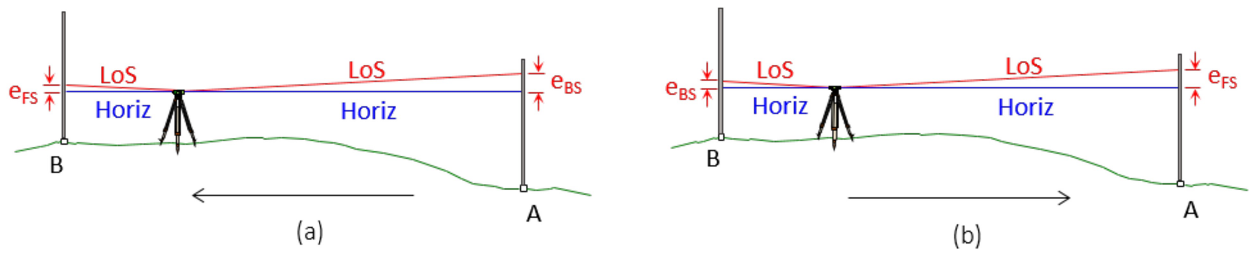


Figure 19: Non-Independent Measurements

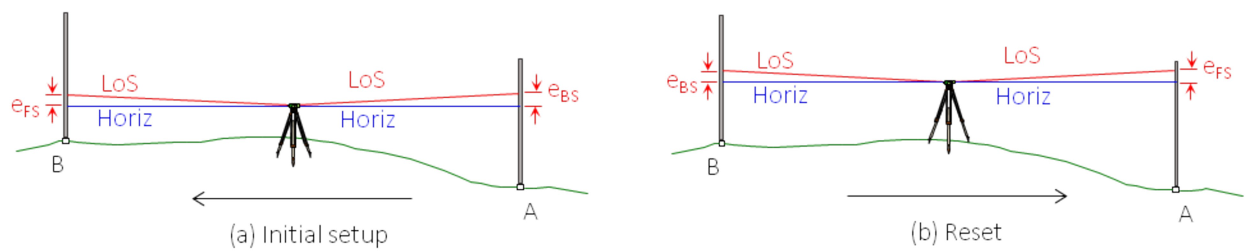


Figure 20: Independent Elevation Measurements

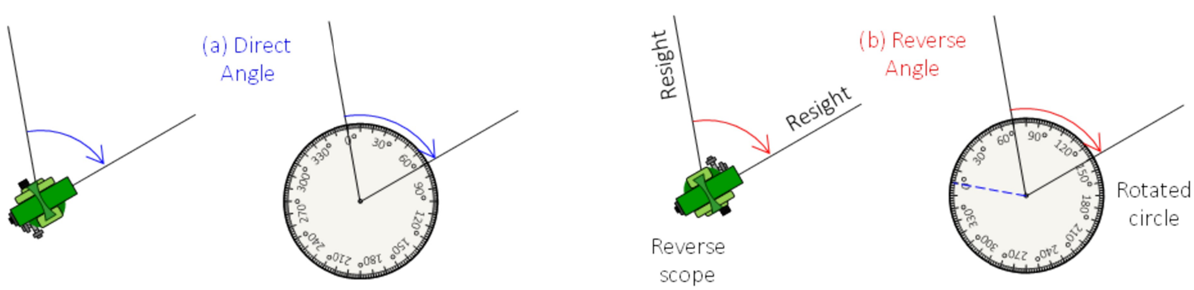


Figure 21: Independent Angle Measurements

Repeating measurements: How many times?

Figure 22 compares 3 different resolution instruments. Each D/R set is two angle measurements. Initial repetition greatest benefit; eventual diminishing returns.

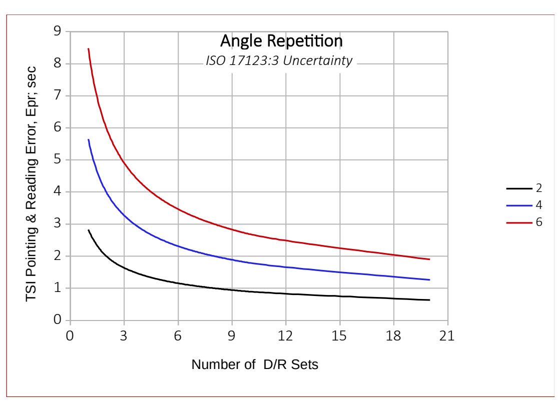


Figure 22: Angle Repetition

III. Random Errors

A. Basic Analysis

Example Measurement set: 45.66 45.66 45.68 45.65

1. Terms

Direct and Indirect Measurements

Redundancy

Discrepancy

Most probable value

$$MPV = \frac{\sum m}{n}$$

Residual

$$v = MPV - m$$

Least Squares

$$\sum(v^2) = min$$

Standard deviation

$$\sigma = \sqrt{\frac{\sum(v^2)}{n-1}}$$

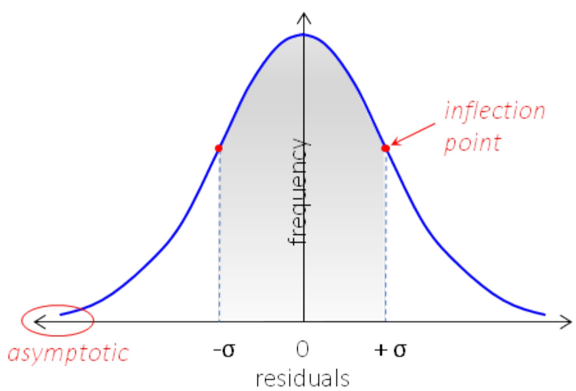


Figure 23: Normal Distribution Curve

e. Confidence interval

f. Standard Error Of the Mean

$$E_{MPV} = \frac{\sigma}{\sqrt{n}}$$

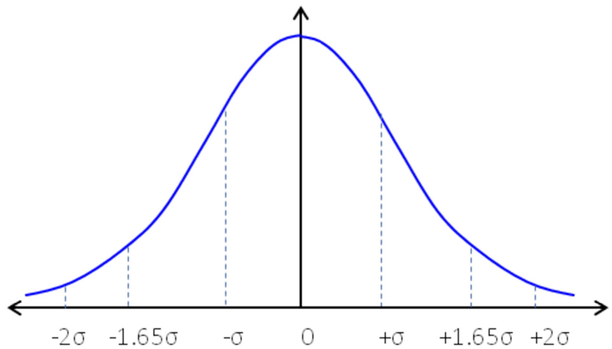


Figure 24: Confidence Intervals

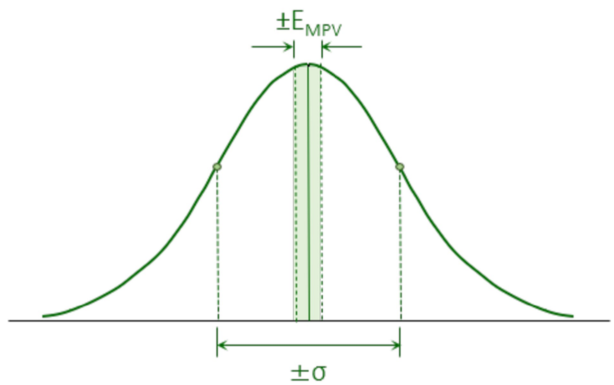


Figure 25: Error of the Mean

2. Example

num	m	v, residual	v^2
1	45.66		
2	45.68		
3	45.66		
4	45.65		
sums:			

3. Weights

Mixing different quality measurements.

$$MPV = \frac{\sum(w \times meas)}{\sum w}$$

Relative weight is inversely proportional to standard deviation squared ($w = 1/s^2$). For distances, it can also be inversely proportional to precision ($w = 1/p$).

Example: A distance was independently measured multiple times by three crews. Their results are in the table.

Crew	Dist (ft)	Std Dev (ft)	Weight	w	w x m
1	351.11	±0.15			
2	351.92	±0.38			
3	352.35	±0.08			
sums					

4. Comparisons

Two survey crews measure different horizontal angles multiple times:

	Crew A	Crew B
Num of meas	2 D/R	4 D/R
Average angle	128°18'15"	196°02'40"
Std Dev	±0°00'12"	±0°00'14"

Which crew has better precision? Why?

Which crew has better expected accuracy? Why?

5. Random Errors Angle Comps

a. Example 1

num	DMS	DD: 3 dp	DD: 7dp	Sec*
1	35°15'30"	35.258°	35.2583333°	30"
2	35°15'56"	35.266°	35.2655556°	56"
3	35°15'35"	35.260°	35.2597222°	35"
4	35°15'47"	35.263°	35.2630556°	47"
	sums	141.047°	141.0466667°	168"
	MPV	35.262°	35.2616667°	42.0"
		35°15'43.2"	35°15'42.0"	+35°15'
				35°15'42.0"
	Std Dev	±0.004°	±0.0032632°	
		±14.4"	±11.7"	±11.7"

* 35°15' is constant for each angle, so subtract it and work with just the seconds.

b. Example 2

num	DMS	Sec	v	v^2
1	107°03'58"			
2	107°04'01"			
3	107°04'03"			
4	107°03'59"			
	sums			

B. Error Propagation

1. Simple

a. Error of a Sum

Adding or subtracting values which are subject to errors.

$$E_{sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

Example:

Soil sample	Weight
1	56.2 gr ±0.15 gr
2	28.9 gr ±0.21 gr
3	41.6 gr ±0.11 gr

Total weight = 126.7 gr ± ______ gr

b. Error of a Series

The same error occurring multiple times.

$$E_{series} = E\sqrt{n}$$

Example: Each time they set up a level, a survey crew can determine an elevation difference to ±0.015 ft. What is their expected misclosure error on this loop?

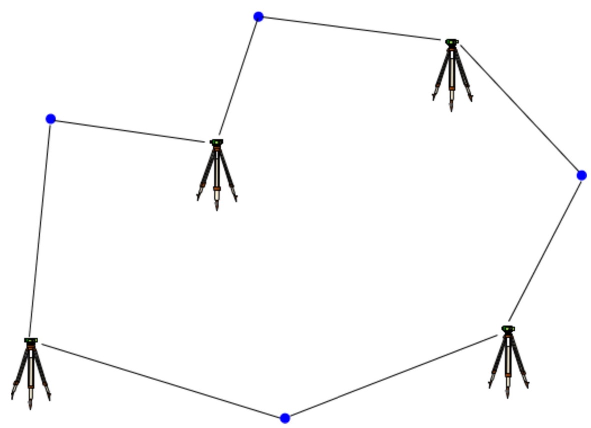


Figure 26: Level Loop Example

c. Error of a Product:

Two values with errors are multiplied or divided.

$A \pm E_A$, $B \pm E_B$

$$E_{prod} = \sqrt{(A \times E_B)^2 + (B \times E_A)^2}$$

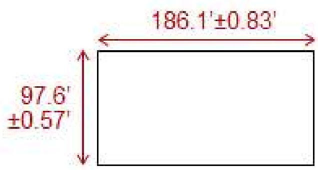


Figure 27. Area Error

2. Combined

a. Distance

$$E_{dist} = \sqrt{E_{si}^2 + C_{msa}^2 + \left(\frac{P_{msa} x D}{1{,}000{,}000}\right)^2 + E_{ref}^2}$$

TSI setup error: ±0.005 ft; handheld reflector centering: ±0.01 ft; 712.36 ft measured distance.

MSA: ±(3 mm + 3 ppm).

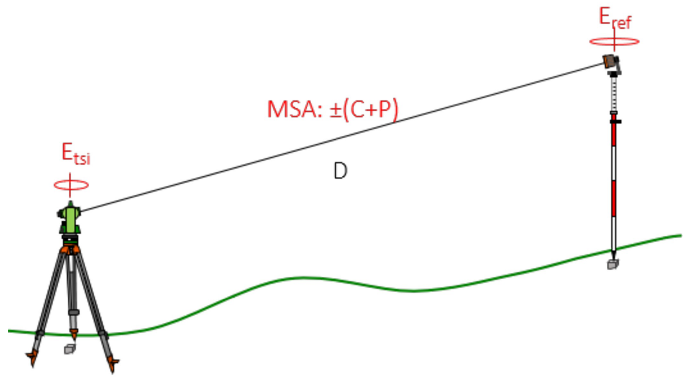


Figure 28. Distance Error

b. Horizontal TSI Angle

$$E_{pr} = \frac{2 \times E_{DIN}}{\sqrt{n}}$$

$$E_{tsi} = \frac{D \times E_i}{D_{BS} D_{FS} \sqrt{2}} \times \frac{206{,}264.8\text{ sec}}{\text{radian}}$$

$$E_t = \sqrt{\left(\frac{E_{BS}}{D_{BS}}\right)^2 + \left(\frac{E_{FS}}{D_{FS}}\right)^2} \times \frac{206{,}264.8\text{ sec}}{\text{radian}}$$

$$E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$$

Angle: 123°30’10” meas’d 2 D/R; TSI DIN: 2 sec; TSI centering error: ±0.005 ft; BS & FS centering errors: ±0.005 ft & ±0.01 ft; BS & FS dists: 176 ft & 243 ft.

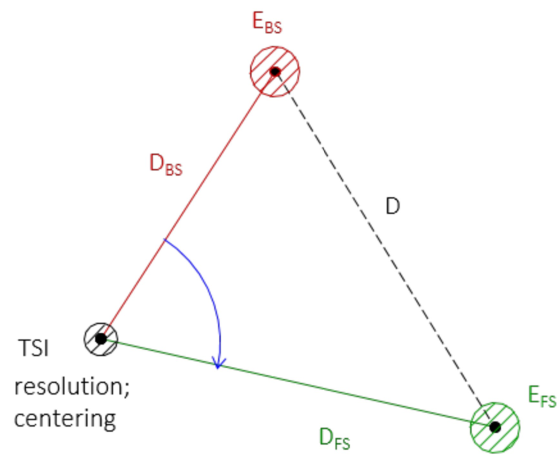


Figure 29. Horizontal Angle Error

IV. Least Squares Adjustment (LSA)

A. Compared to Simplified Adjustments

1. Advantages

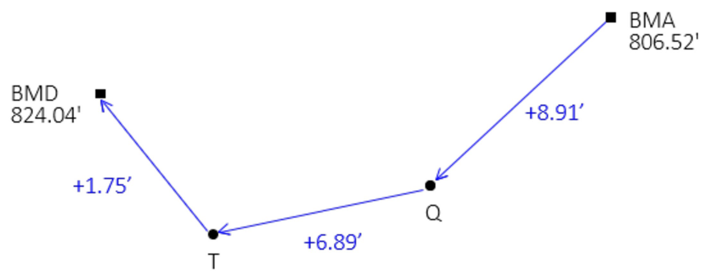
Better modeling of random error behavior
Able to deal with multiple unknowns simultaneously
Easily incorporates redundant measurements - the more the merrier
Can use mixed quality measurements
Can generate statistics for overall adjustment and individual solved unknowns

2. Disadvantages

Computation intensive
Statistics overload
Easy to misuse

B. Building Complex Networks

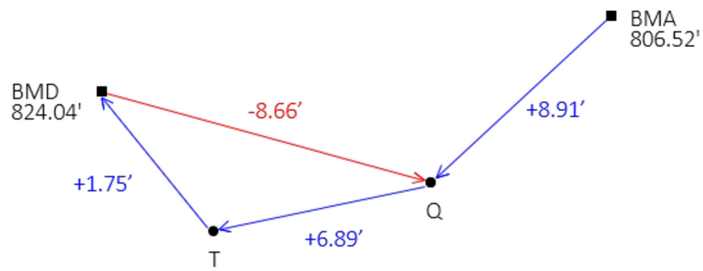
Multiple unknowns



Can apply simple network adjustment.

Figure 30: Three observations

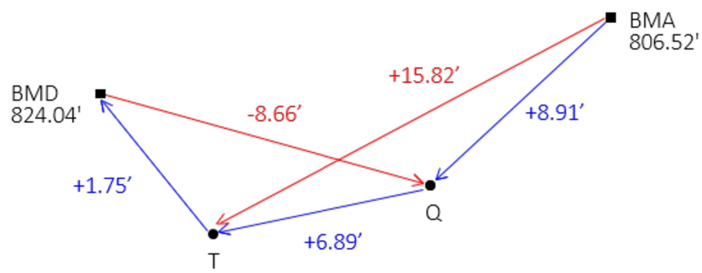
Add fourth observation.



Simple network adjustment is cumbersome.

Figure 31: Fourth Observation Added

Add fifth observation.



Simple network adjustment not possible.

Figure 32: Fifth Observation Added

C. Random Errors Only

Mistakes and systematic errors must be eliminated/compensated.

Mistake - effect can be spread into other measurements, Figure 33

Systematic errors - residual sizes and distribution depend on:

Error character - scale or additconstant

Which measurements are affected

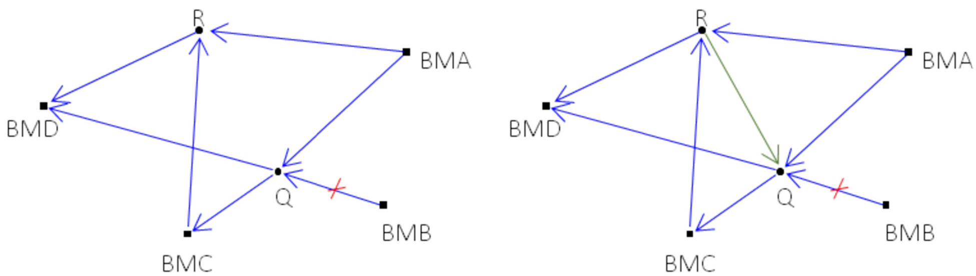


Figure 33: Mistake Inclusion

D. Redundancies

1. General

Redundancy aka degree of freedom (df): df = m-n

m: number of measurements
n: number of unknowns

If...	Then...
m<n	
m=n	
m>n	

2. Vertical Network - 1D

How many observations are necessary to determine the elevations of points A and B from the benchmark, BM Dog, Figure 34?

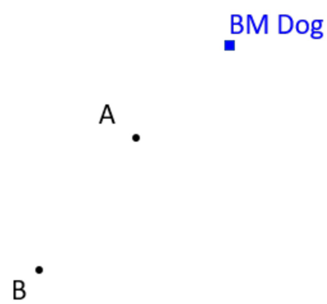


Figure 34: Elevation Points

How many df in

Figure 35?
Figure 36?
Figure 37?

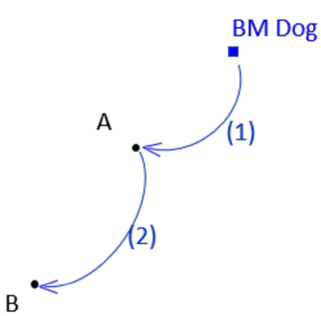


Figure 35: 2 Observations

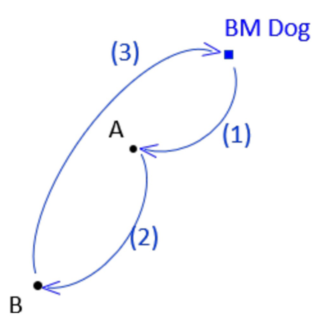


Figure 36: 3 Observations

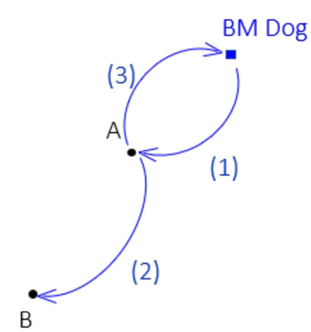


Figure 37: 3 Observations

Difference between Figures 36and 37?

Figure 36 - true redundancies
Figure 37- pseudo-redundancies

3. Horizontal Network - 2D

Unknowns are point positions: coordinates.

How many unknowns in Figure 38?

Traditional traverse, Figure 39:

Fix one point and one direction.
Measure sides and angles between
Num df?

Can apply simple network adjustment (eg, Compass Rule, Transit Rule).

Add redundancies?

Fix one point and one direction; Measure sides and angles between.

Num df in
- Figure 40?
- Figure 41?
- Figure 42?

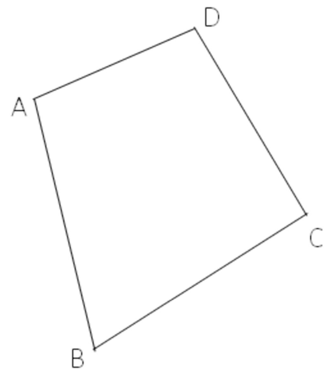


Figure 38: Lot Corners

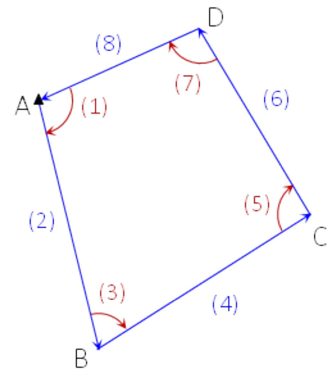


Figure 39: Traverse

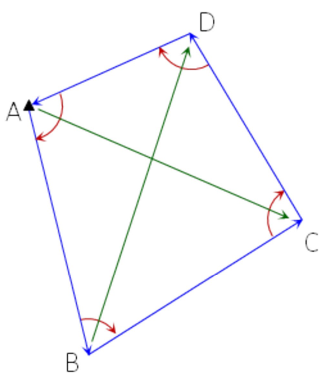


Figure 40: Add Diagonals

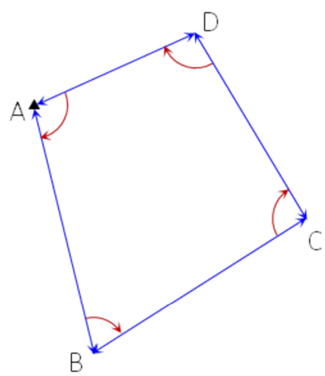


Figure 41: Distances in Both Directions; Angles

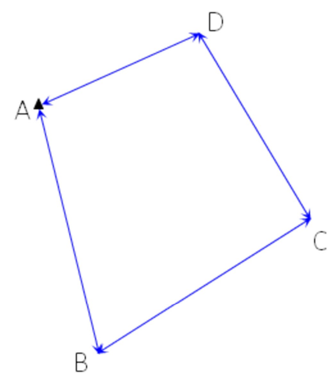


Figure 42: Distances Only

True vs pseudo-redundancies

D. Weights

Sometimes science, sometimes art

1. Vertical - Leveling

Distances
Number of setups

2. Horizontal

Distances

$$E_{dist} = \sqrt{E_{si}^2 + C_{msa}^2 + \left(\frac{P_{msa} x D}{1,000,000}\right)^2 + E_{ref}^2}$$

Directions and Angles

$$E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$$

E. Linear/Non-Linear

An LSA minimizes the sum of the squares of the residuals of the observations: $\Sigma(v^2)$=min.

1. Vertical - Linear

Measurements are simple addition & subtraction: $Elev_B = Elev_A + BS_A - FS_B$

Non-iterative solution.

Example: For each line, write an observation equation which includes a residual term.

$$Elev_Q = 806.52 + 8.91 + v_{aq}$$
$$Elev_T = Elev_Q + 6.89 + v_{qt}$$
$$824.04 = Elev_T + 1.75 + v_{tb}$$
$$Elev_Q = 824.04 - 8.66 + v_{bq}$$

$$v_{aq} = Elev_Q - 815.43$$
$$v_{qt} = Elev_T - Elev_Q - 6.89$$
$$v_{tb} = 822.29 - Elev_T$$
$$v_{bq} = Elev_Q - 815.38$$

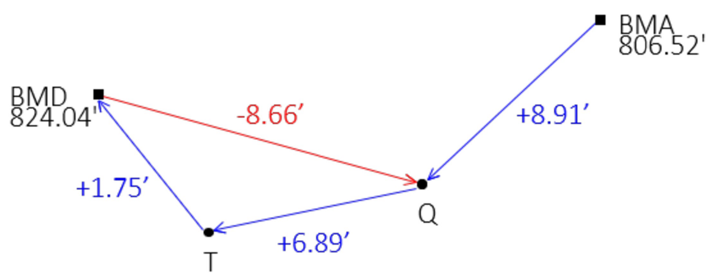


Figure 43: Level Network Adjustment

Square the residuals and add them.

$$F = \sum_1^n v_i^2 = \left(Elev_Q - 815.43\right)^2 + \left(Elev_T - Elev_Q - 6.89\right)^2 + \left(822.29 - Elev_T\right)^2 + \left(Elev_Q - 815.38\right)^2$$

To minimize residuals, the partial derivative of the function with respect to each unknown elevation is set equal to 0.

$$\frac{\partial F}{\partial Elev_Q} = 2(1)\left(Elev_Q - 815.43\right) + 2(-1)\left(Elev_T - Elev_Q - 6.89\right) + 2(1)\left(Elev_Q - 815.38\right) = 0.000$$
$$\cdots \Rightarrow 3\left(Elev_Q\right) - Elev_T - 1623.92 = 0.000$$

$$\frac{\partial F}{\partial Elev_T} = 2(1)\left(Elev_T - Elev_Q - 6.89\right) + 2(-1)\left(822.29 - Elev_T\right) = 0.000$$
$$\cdots \Rightarrow 2\left(Elev_T\right) - Elev_Q - 829.18 = 0.000$$

The two equations can be solved simultaneously for the unknown elevations.

$Elev_Q$ = 815.404', $Elev_T$ = 822.292'

2. Horizontal - Non-Linear

Measurements are angles and distances.

Position determination requires trig which is non-linear.

LSA solution uses coordinate variation

Start with approximate coordinates

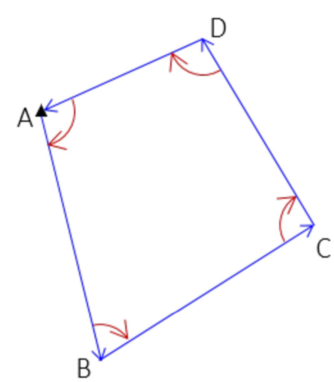


Figure 44: Horizontal

Compute and apply corrections
Repeat until corrections are below a threshold

a. Measurements

(1) Distances

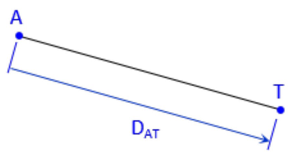


Figure 45: Distance Measurement

The residual is the difference between the measured distance and the distance computed from coordinates.

$$v_{at} = D_{AT} - \left[(N_T - N_A)^2 + (E_T - E_A)^2 \right]^{1/2}$$

Four partial derivatives (one for each coordinate) of the residual squared:

$$\frac{\partial D_{AT}}{\partial N_A} = \left[\frac{-(N_T - N_A)}{D_{AT}} \right] dN_A \qquad \frac{\partial D_{AT}}{\partial E_A} = \left[\frac{-(E_T - E_A)}{D_{AT}} \right] dE_A$$

$$\frac{\partial D_{AT}}{\partial N_T} = \left[\frac{(N_T - N_A)}{D_{AT}} \right] dN_T \qquad \frac{\partial D_{AT}}{\partial E_T} = \left[\frac{(E_T - E_A)}{D_{AT}} \right] dE_T$$

dN_A, dE_A, dN_T, dE_T are corrections for the initial coordinates.

(2) Angles and Directions

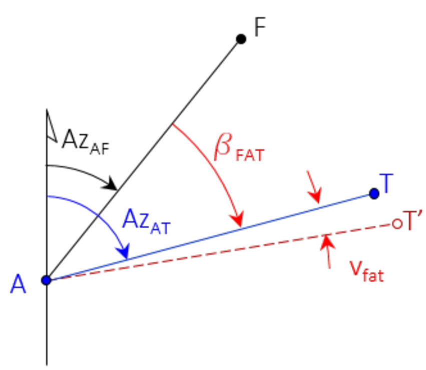


Figure 46: Angle Measurement

The residual is the difference between the measured angle and the angle computed from coordinates.

$$v_{fat} = \beta_{FAT} - (Az_{AT} - Az_{AF}) = \beta_{FAT} - \left(\tan^{-1}\left[\frac{E_T - E_A}{N_T - N_A} \right] - \tan^{-1}\left[\frac{E_F - E_A}{N_F - N_A} \right] \right)$$

Six partial derivatives (one for each coordinate) of the residual squared:

$$\frac{\partial \beta_{FAT}}{\partial E_N} = \left[\frac{(E_T - E_A)}{D_{AT}^2} - \frac{(E_F - E_A)}{D_{AF}^2} \right]_O dN_A \qquad \frac{\partial \beta_{FAT}}{\partial E_A} = \left[\frac{-(N_T - N_A)}{D_{AT}^2} - \frac{-(N_F - N_A)}{D_{AF}^2} \right] dE_A$$

$$\frac{\partial \beta_{FAT}}{\partial N_F} = \left[\frac{(E_F - E_A)}{D_{AF}^2} \right] dN_F \qquad \frac{\partial \beta_{FAT}}{\partial E_F} = \left[\frac{-(N_F - N_A)}{D_{AF}^2} \right] dE_F$$

$$\frac{\partial \beta_{FAT}}{\partial N_T} = \left[\frac{-(E_T - E_A)}{D_{AT}^2} \right] dN_T \qquad \frac{\partial \beta_{FAT}}{\partial E_T} = \left[\frac{(N_T - N_A)}{D_{AT}^2} \right] dE_T$$

dN_A, dE_A, dN_F, dE_F, dN_T, dE_T are corrections for the initial coordinates.

b. Iterative

Requires initial coordinate approximations

Corrections are combined at points common to angles and distance measurements.

Iterates correcting coordinates until acceptable level is reached.

Needless to say, this can be quite a lot of calculations which is why we use software.

F. Adjustment Statistics

1. Overall

Standard Deviation of Unit Weight (S_O): $$S_o = \sqrt{\frac{\sum_{i=1}^{m}\left(v_i^2\right)}{(m-n)}}$$

2. Unknowns

a. Vertical

S_{Elev}

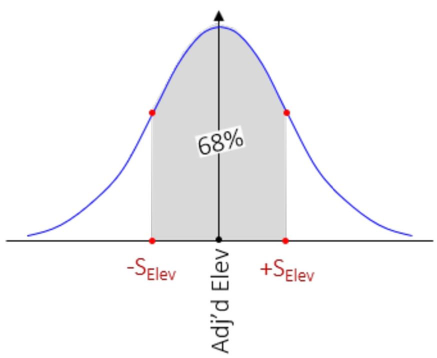


Figure 47: Adjusted Elevation

b. Horizontal

S_N & S_E

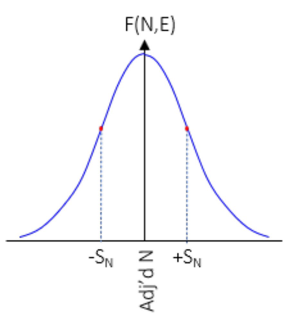


Figure 48: Adjusted North Coordinate

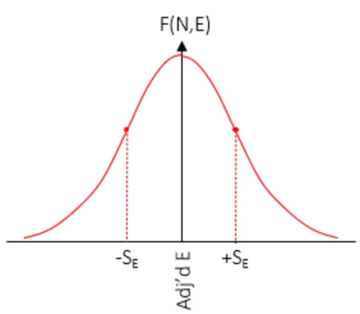


Figure 49: Adjusted East Coordinate

Bi-variate; Standard Error Ellipse

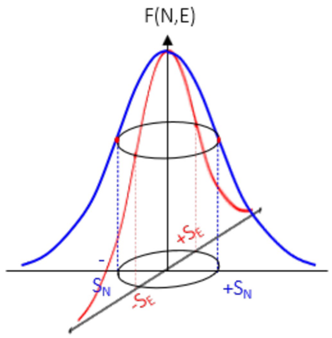


Figure 50: Bi-Variate Distribution

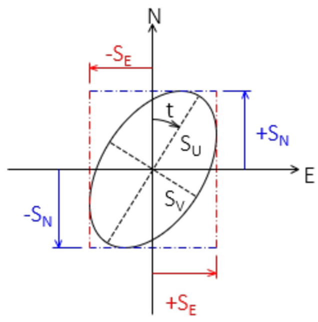


Figure 51: Standard Error Ellipse

Standard error ellipse is ~33-35% CI

Confidence Intervals

CI = f(df): $S_{i@CI\%} = S_i \times \sqrt{2 \times F}$

@95% CI:

DF	F	DF	F
1	199.5	4	6.94
2	19.0	5	5.79
3	9.55	10	4.10

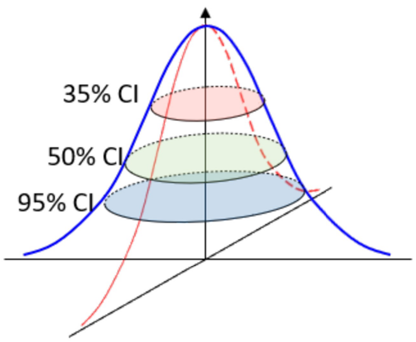


Figure 52: Different CI

c. Adjustment Considerations

(1) Adjacent Adjustments

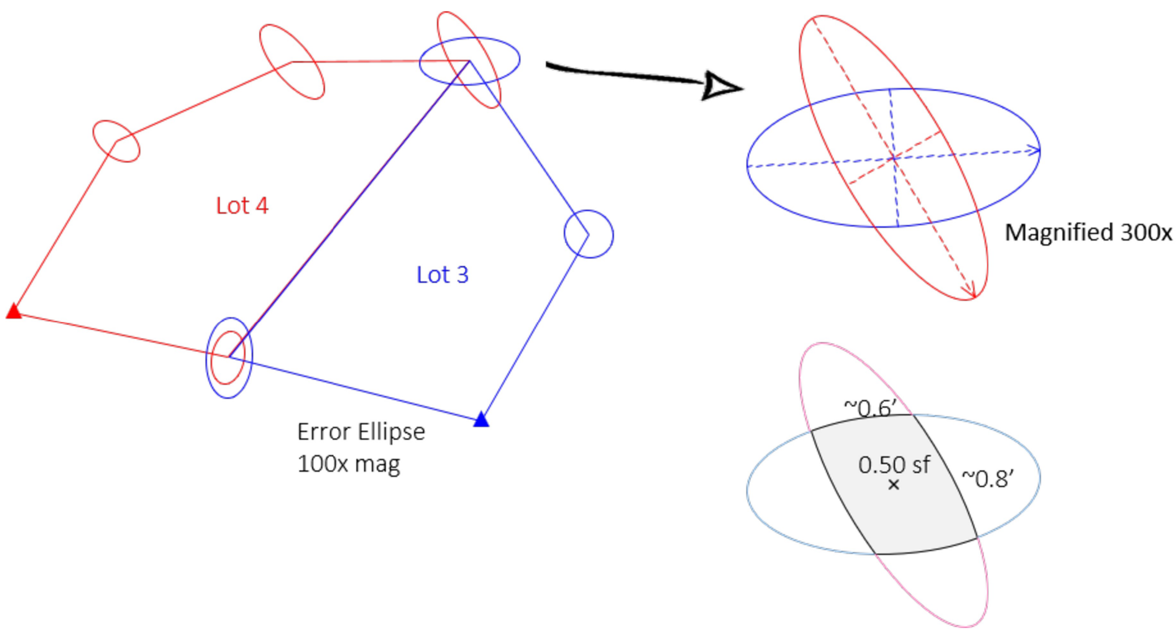


Figure 53: Overlapping Error Ellipses

Pin-cushions (monument nests), Figure 54?

Figure 54: Pin-Cushions

(2) Noise level?

Are some adjusted values significant or not, Figure 55?

Commencing at a 2" iron pipe at the west quarter corner of Section 31, T5N, R10E;
Thence S88°16'52"W, 0.30 feet to the existing east line of Section 1, T5N, R9E;
Thence S00°18'01"W, 0.01 feet along said east line of said Section 1;
Thence S00°18'01"W, 33.20 feet along said east line;
Thence N88°34'15"E, 33.78 feet to the existing east right-of-way line of STH 104, also being the point of beginning;
Thence N88°47'53"E, 803.03 feet along the existing south right-of-way line of STH 92;
Thence N88°17'20"E, 55.46 feet along said south right-of-way line;

Figure 55: Statistically Significant?