I. Useful Equations and Relationships

A. Triangles

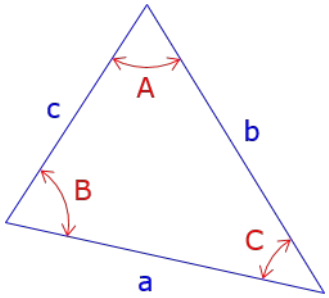


Figure 1 Triangle

Angle condition $A + B + C = 180^\circ 00' 00''$

Law of Sines $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Law of Cosines $a^2 = b^2 + c^2 - 2 \times (bc) \times \cos(A)$

$b^2 = c^2 + a^2 - 2 \times (ca) \times \cos(B)$

$c^2 = a^2 + b^2 - 2 \times (ab) \times \cos(C)$

Area $\frac{1}{2}(ab) \times \sin(C)$

$\frac{1}{2}(bc) \times \sin(A)$

$\frac{1}{2}(ca) \times \sin(B)$

$\sqrt{s \times (s-a) \times (s-b) \times (s-c)}$

$s = \frac{a+b+c}{2}$

B. Circles and Arcs

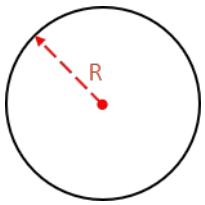


Figure 2 Circle

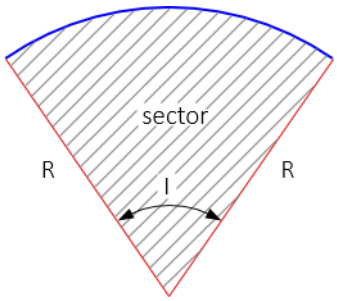


Figure 3 Sector

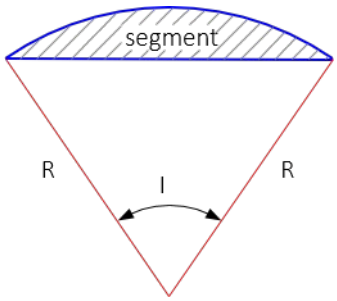


Figure 4 Segment

$\text{Area} = \pi R^2$

$\text{Perimeter} = 2\pi R$

$\text{Area} = \frac{l \times \pi \times R^2}{360^\circ}$

$\text{Area} = R^2 \left(\frac{\pi \times l}{360^\circ} - \frac{\sin(l)}{2} \right)$

C. Circular Curve

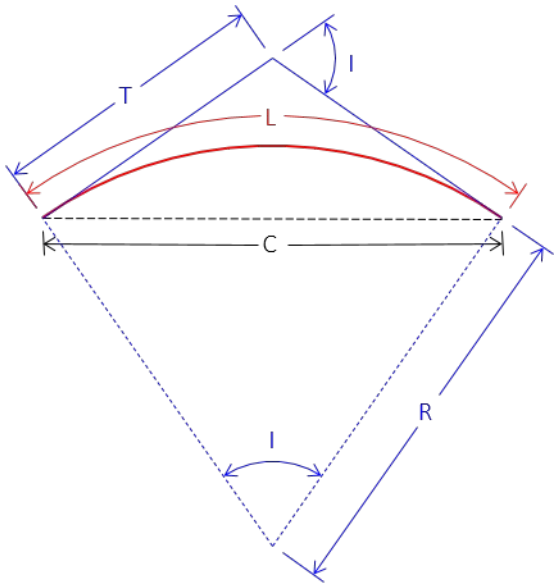


Figure 5 Circular Curve

$$L = \frac{I \times \pi \times R}{180^\circ} = 100 \times \left(\frac{I}{D}\right)$$

$$R = \frac{5729.58}{D}$$

$$C = 2R \times \sin\left(\frac{I}{2}\right)$$

$$T = R \times \tan\left(\frac{I}{2}\right)$$

R : radius; ft
L : arc length; ft
C : chord length; ft
T : tangent length; ft
D : degree of curvature,
arc definition; degrees
I : central angle; degrees

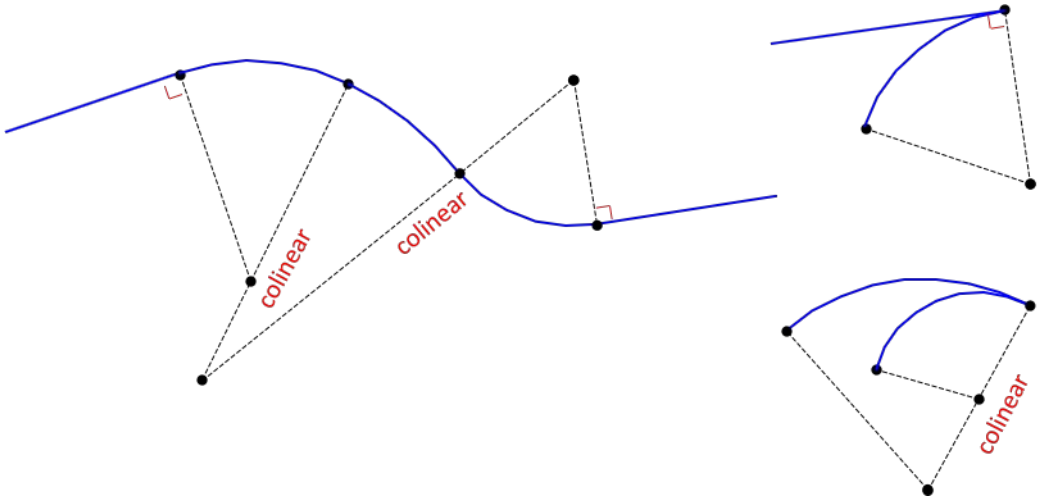


Figure 6 Tangency Conditions

D. Coordinates

Forward computation

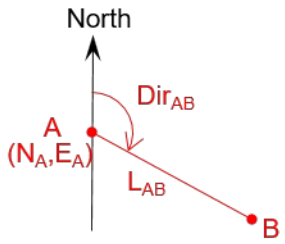


Figure 7 Forward

$$N_B = N_A + Lat_{AB}$$
$$= N_A + L_{AB} \times \cos(Dir_{AB})$$
$$E_B = E_A + Dep_{AB}$$
$$= E_A + L_{AB} \times \sin(Dir_{AB})$$

Inverse computation

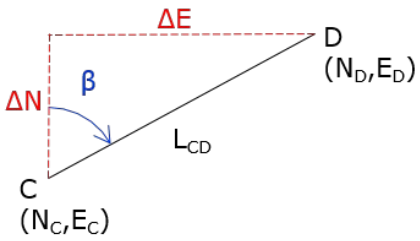


Figure 8 Inverse

$$\Delta N = N_D - N_C$$
$$\Delta E = E_D - E_C$$
$$L_{CD} = \sqrt{\Delta N^2 + \Delta E^2}$$
$$\beta = \tan^{-1}\left[\frac{\Delta E}{\Delta N}\right]$$

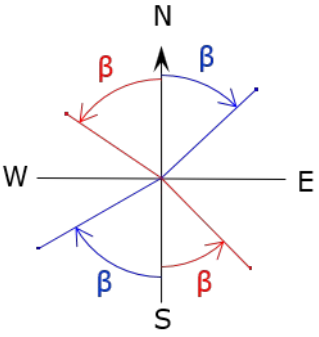
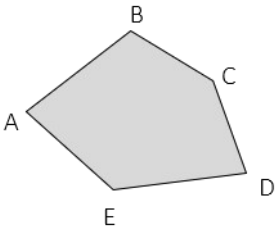


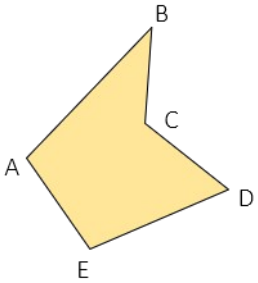
Figure 9 Meridian Angle

Algebraic sign

Quad	*ΔN*	*ΔE*	*β*	*Bearing*	*Azimuth*
NE	+	+	+	N β E	β
SE	–	+	–	S | β | E	180°+ β
SW	–	–	+	S β W	180°+ β
NW	+	–	–	N | β| W	360°+ β

Area





		(↗)	(↘)
N_A	E_A	$N_B \times E_A$	
N_B	E_B	$N_C \times E_B$	$N_A \times E_B$
N_C	E_C	$N_D \times E_C$	$N_B \times E_C$
N_D	E_D	$N_E \times E_D$	$N_C \times E_D$
N_E	E_E	$N_A \times E_E$	$N_D \times E_E$
N_A	E_A		$N_E \times E_A$
sums:		$\Sigma(\nearrow)$	$\Sigma(\searrow)$

$$Area = \frac{\Sigma(\nearrow) - \Sigma(\searrow)}{2}$$

Figure 10 Areas

II. Example Problems

Problem A[1]

The centerline of a curvilinear street must be relocated, Figure 11. The center of the second curve will be shifted N40°24’00”W 30.00 ft (O_2 to $O_{2'}$), keeping its 173.00 ft radius. The centers of the other curves will not move, but their curve geometries will change.

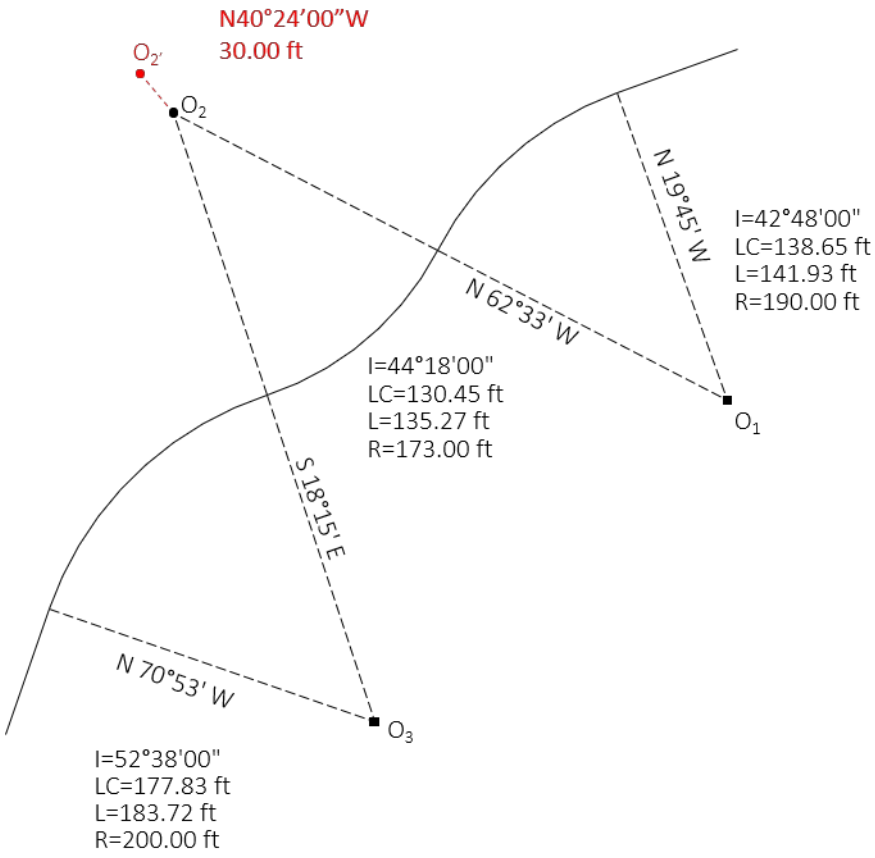


Figure 11 - Original Alignment

Select the correct triplet of answers for the following curve parts.

1 Problems A and B are from J. Van Sickle’s *Surveying Solved Problems*.

Part A.1 Central angles.

	Curve 1	*Curve 2*	*Curve 3*
a.	41°08’30”	41°01’24”	51°00’54”
b.	41°08’08”	42°00’15”	53°38’23”
c.	42°35’39”	44°18’25”	51°52’02”
d.	42°40’15”	40°31’30”	52°35’50”

Part A.2 Curve radii.

	Curve 1	*Curve 2*	*Curve 3*
a.	190.00 ft	173.00 ft	200.00 ft
b.	190.00 ft	200.93 ft	200.00 ft
c.	204.62 ft	186.31 ft	214.98 ft
d.	217.95 ft	173.00 ft	227.96 ft

Part A.3 Long chord lengths.

	Curve 1	*Curve 2*	*Curve 3*
a.	153.16 ft	121.24 ft	196.33 ft
b.	154.66 ft	123.88 ft	202.96 ft
c.	155.32 ft	115.69 ft	202.96 ft
d.	155.48 ft	135.27 ft	135.27 ft

Part A.4 Arc lengths.

	Curve 1	*Curve 2*	*Curve 3*
a.	143.67 ft	127.37 ft	231.07 ft
b.	153.14 ft	121.25 ft	196.33 ft
c.	155.03 ft	128.93 ft	200.86 ft
d.	156.50 ft	123.87 ft	202.97 ft

Part A.5 Long chord bearings

	Curve 1	*Curve 2*	*Curve 3*
a.	S48°51’00”W	S49°36’00”W	S45°26’00”W
b.	S49°15’54”W	S49°36’51”W	S45°01’48”W
c.	S49°40’45”W	S49°37’18”W	S44°37’44”W
d.	S49°51’25”W	S49°26’09”W	S44°59’43”W

Problem B

The circular curve shown in Figure 12 encroaches onto Lot 4. It will be replaced by a compound curve tangent to the northern boundary of Lot 4, Figure 13. The existing PC will be the start of the first curve, point A will serve as its PI, and point C will be the PI of the second curve.

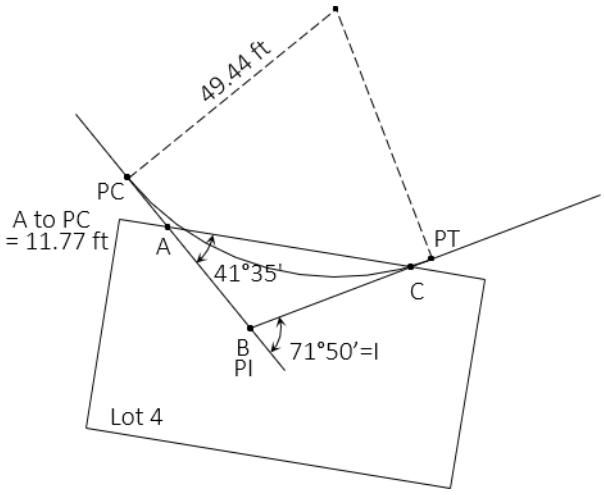


Figure 12 - Single Curve

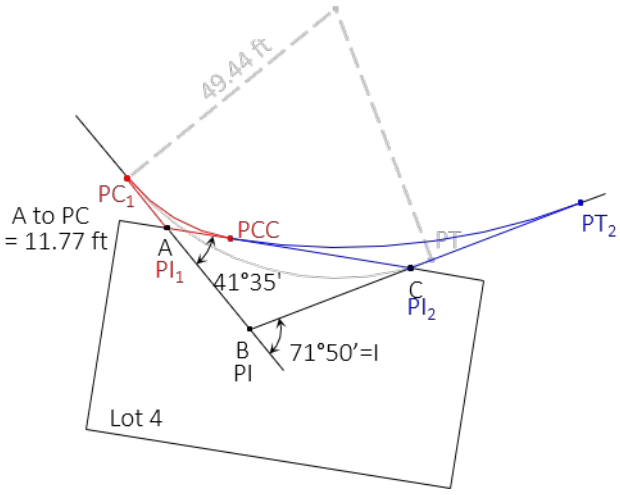


Figure 13 - Compound Curve

Part B.1 What is the radius of the first curve?

a. 31.00 ft
b. 47.35 ft
c. 49.44 ft
d. 52.33 ft

Part B.2 What is the radius of the second curve?

a. 121.35 ft
b. 122.30 ft
c. 123.25 ft
d. 124.20 ft

Part B.3 What are the long chord lengths?

a. 20.18 ft and 50.43 ft
b. 20.18 ft and 64.21 ft
c. 22.01 ft and 64.81 ft
d. 23.89 ft and 64.81 ft

Problem C

A surveyor set up her total station atop a forest lookout tower on a hill overlooking a river. With an instrument height of 5.2 ft, she measured a 115°22'06" zenith angle to a point on the far side of the river where the water intersected the shore and 129°15'49" to a point on the near side. The two points and instrument location were all in the same vertical plane. Later, back on terra firma, she measured the horizontal distance as 225.35 ft between the two river points. If the river elevation is 841.2 ft what is the elevation, to 0.1 ft, of the lookout tower top?

a. 954.4 ft
b. 1090.4 ft
c. 1095.6 ft
d. 1243.2 ft

Problem D

A surveyor set up her total station instrument (TSI) at location from which she could see three corners of a square parcel. She measured horizontal distance to the three corners as shown in Figure 14. What is the parcel's area to the nearest 10 square feet?

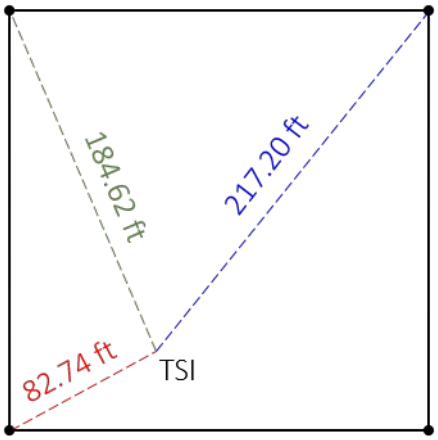


Figure 14 - Lot Area

a. 15,520 sq ft
b. 28,060 sq ft
c. 35,450 sq ft
d. 43,530 sq ft

Problem E

A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent, Figure 15. The data shown is for the existing reverse curve situation.

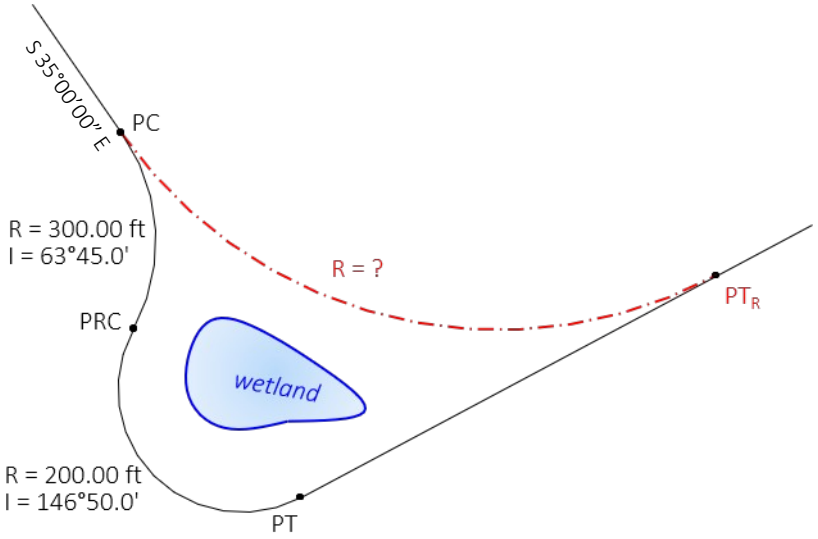


Figure 15 - Replacement Curve

What is the radius, to the nearest 0.1 ft, of the replacement single curve?

a. 500.0 ft
b. 653.2 ft
c. 744.3 ft
d. 895.2 ft

What is the distance, to the nearest 0.1 ft, along the tangent from the old PT to the new PT_2?

a. 536.4 ft
b. 601.2 ft
c. 688.0 ft
d. 763.2 ft

Problem F

To the nearest 10 square feet, what is the area between the two tangent circular arcs and tangent lines in Figure 16.

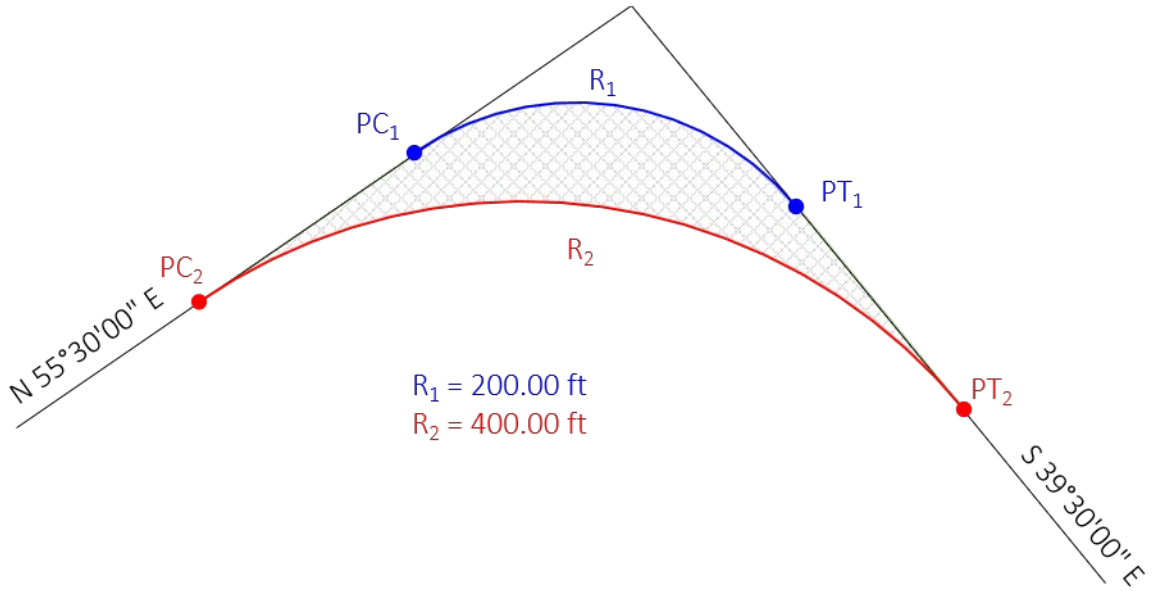


Figure 16 - Area Between Curves

a. 20,950 sf
b. 29,770 sf
c. 118,680 sf
d. 139,630 sf