



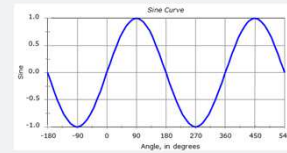
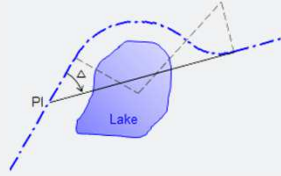
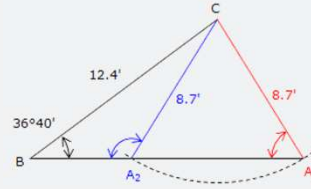
Math Problem Examples

Mentoring Mondays, 1 Nov 2021

Jerry Mahun, PLS

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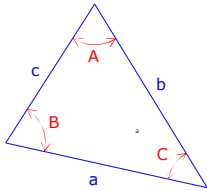
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Useful Equations and Relationships Triangles



Angle condition $A + B + C = 180^{\circ}00'00''$

Law of Sines $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Law of Cosines $a^2 = b^2 + c^2 - 2 \times (bc) \times \cos(A)$
 $b^2 = c^2 + a^2 - 2 \times (ca) \times \cos(B)$
 $c^2 = a^2 + b^2 - 2 \times (ab) \times \cos(C)$

Area $\frac{1}{2}(ab) \times \sin(C)$
 $\frac{1}{2}(bc) \times \sin(A)$
 $\frac{1}{2}(ca) \times \sin(B)$

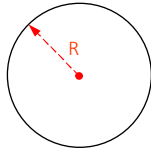
$$\sqrt{s \times (s - a) \times (s - b) \times (s - c)}$$

$$s = \frac{a + b + c}{2}$$

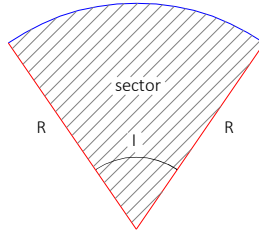
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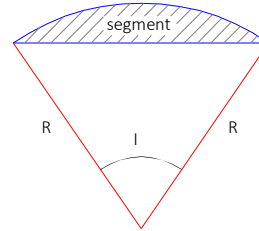
Useful Equations and Relationships
 Circles and Arcs



Area = πR^2
 Perimeter = $2\pi R$
 where R is the circle radius.



$$\text{Area} = \frac{I \times \pi \times R^2}{360^\circ}$$

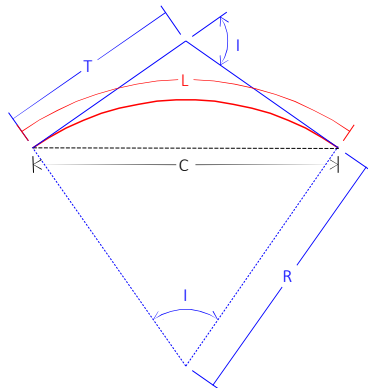


$$\text{Area} = R^2 \left(\frac{\pi \times I}{360^\circ} - \frac{\sin(I)}{2} \right)$$

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Useful Equations and Relationships
 Circular Curves



$$L = \frac{I \times \pi \times R}{180^\circ} = 100 \times \left(\frac{I}{D} \right)$$

$$R = \frac{5729.58}{D}$$

$$C = 2R \times \sin\left(\frac{I}{2}\right)$$

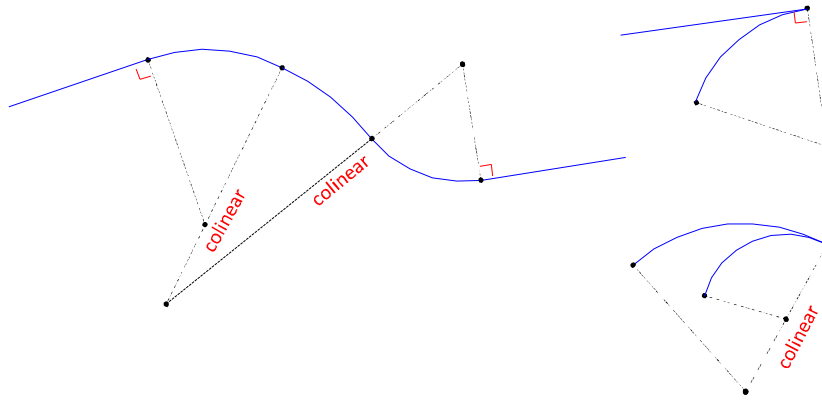
$$T = R \times \tan\left(\frac{I}{2}\right)$$

- R : radius; ft
- L : arc length; ft
- C : chord length; ft
- T : tangent length; ft
- D : degree of curvature,
arc definition; degrees
- I : central angle; degrees

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Useful Equations and Relationships
Circular Curves – Tangency
Perpendicular and colinear conditions



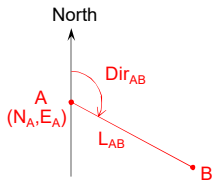
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Useful Equations and Relationships
Coordinates

Inverse Computation

Forward Computation

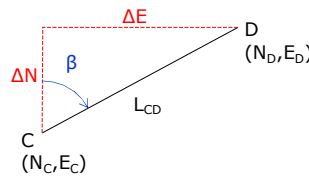


$$N_B = N_A + Lat_{AB}$$

$$= N_A + L_{AB} \times \cos(Dir_{AB})$$

$$E_B = E_A + Dep_{AB}$$

$$= E_A + L_{AB} \times \sin(Dir_{AB})$$

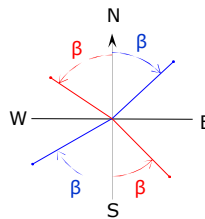


$$\Delta N = N_D - N_C$$

$$\Delta E = E_D - E_C$$

$$L_{CD} = \sqrt{\Delta N^2 + \Delta E^2}$$

$$\beta = \tan^{-1} \left[\frac{\Delta E}{\Delta N} \right]$$



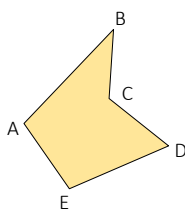
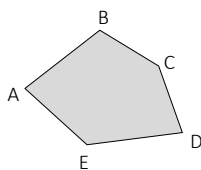
Quad	Algebraic sign			Bearing	Azimuth
	ΔN	ΔE	β		
NE	+	+	+	N β E	β
SE	-	+	-	S β E	$180^\circ + \beta$
SW	-	-	+	S β W	$180^\circ + \beta$
NW	+	-	-	N β W	$360^\circ + \beta$



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Useful Equations and Relationships Coordinates: Area computations



	(\nearrow)	(\searrow)
$N_A \rightarrow E_A$	$N_B \times E_A$	
$N_B \rightarrow E_B$	$N_C \times E_B$	$N_A \times E_B$
$N_C \rightarrow E_C$	$N_D \times E_C$	$N_B \times E_C$
$N_D \rightarrow E_D$	$N_E \times E_D$	$N_C \times E_D$
$N_E \rightarrow E_E$	$N_A \times E_E$	$N_D \times E_E$
$N_A \rightarrow E_A$		$N_E \times E_A$
sums:	$\Sigma(\nearrow)$	$\Sigma(\searrow)$

$$\text{Area} = \frac{\Sigma(\nearrow) - \Sigma(\searrow)}{2}$$



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Rounding may be an issue depending on how many significant figures are shown in the problem and carried in the calculations (yours and the authors).

It's usually safest to carry additional decimal places in your comps

You may have to pick the answer that's closest to what you compute. The difference is probably a reflection of the number of digits carried in computations.

On questions with multiple choice answers, you can sometimes (not always) use logic/reasoning to narrow the choices.

Can one or more answers be eliminated because they "behave" incorrectly?

Example: taping distance in cold temps, the tape shrinks so recorded measurements are *too long*. Toss the answers that are *too short*.

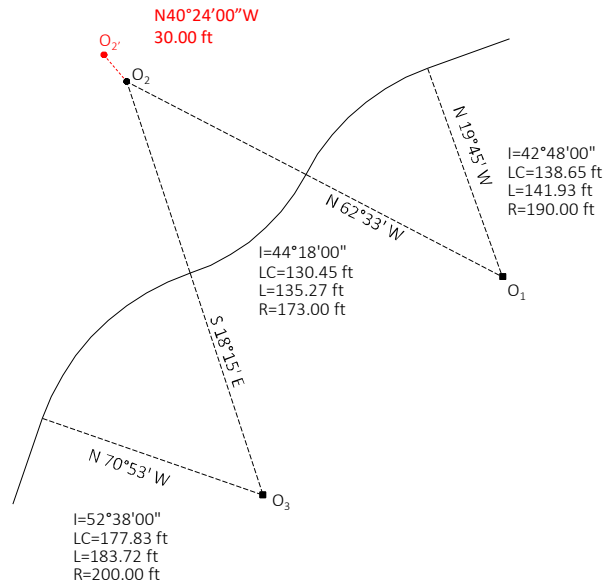
In a time crunch, logical analysis might help increase odds of "guessing" correctly.



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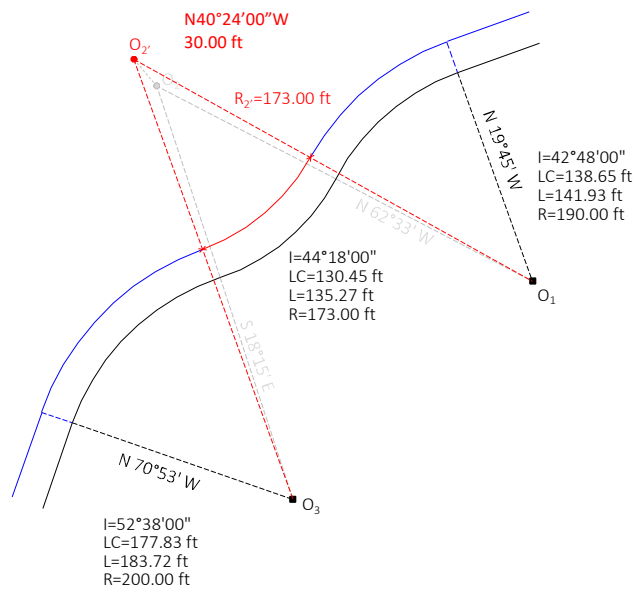
Problem A. The centerline of a street must be relocated. The center of the second curve will be shifted N40°24'00"W 30.00 ft, keeping its 173.00 ft radius. The centers of the other curves will not move, but their curve geometries will change.



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Problem A. The centerline of a street must be relocated. The center of the second curve will be shifted N40°24'00"W 30.00 ft, keeping its 173.00 ft radius. The centers of the other curves will not move, but their curve geometries will change.

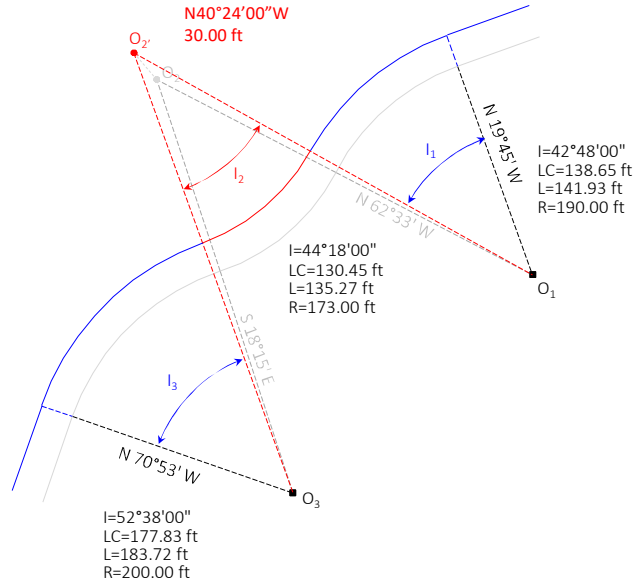


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Part A.1 What are the new central angles for the three curves?

	Curve 1	Curve 2	Curve 3
a.	41°08'30"	41°01'24"	51°00'54"
b.	41°48'08"	42°00'15"	53°38'23"
c.	42°35'39"	44°18'25"	51°52'02"
d.	42°40'15"	40°31'00"	52°35'50"



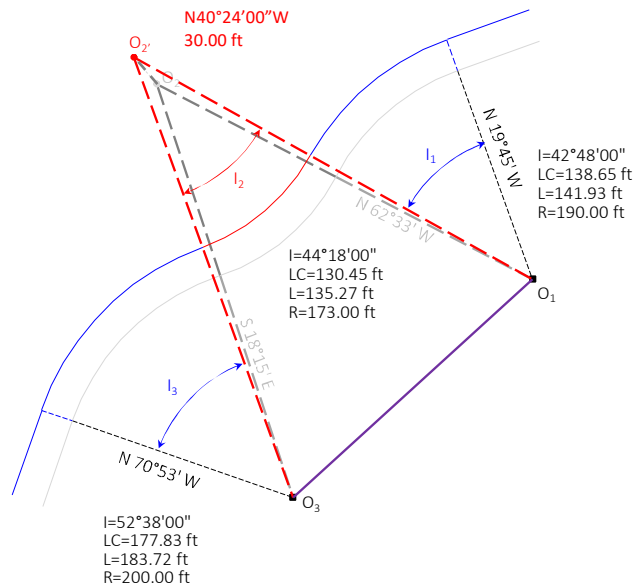
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Part A.1 What are the new central angles for the three curves?

Solution logic?

We could solve multiple triangles.



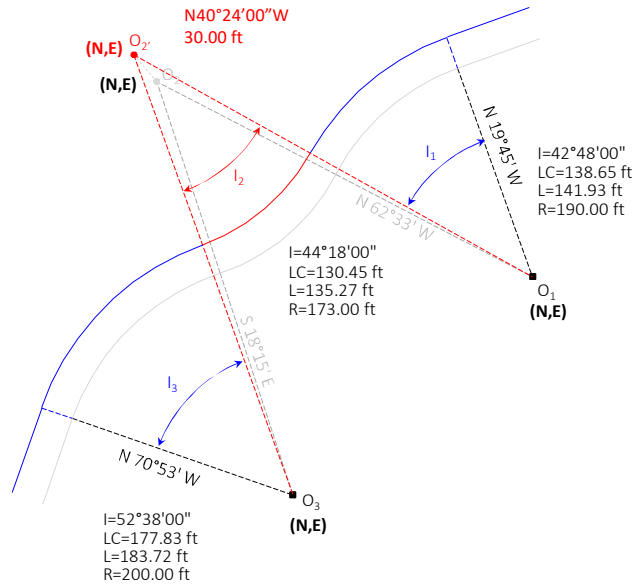
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Part A.1 What are the new central angles for the three curves?

Solution logic?

We could solve multiple triangles.
Or use COGO: I-angles are between directions



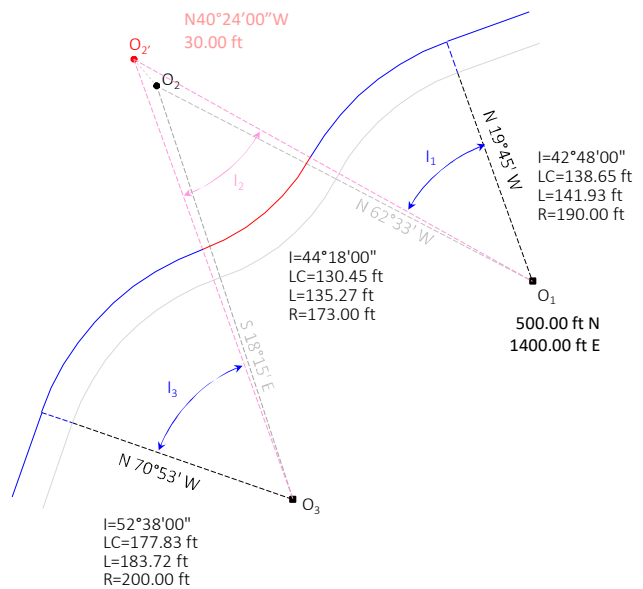
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Part A.1 What are the new central angles for the three curves?

Using COGO

Start at O_1 using 500.00 ft N, 1400.00 ft E (assumed)



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Part A.1 What are the new central angles for the three curves?

Using COGO

Forward Comp to O₂:

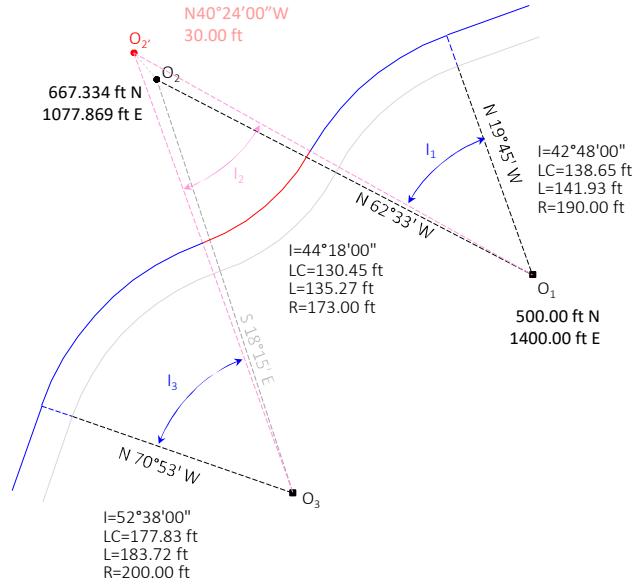
$$Az = 360^{\circ}00'00'' - 62^{\circ}33'00'' = 297^{\circ}27'00''$$

$$N = 500.00\text{ft} + (190.00\text{ft} + 173.00\text{ft}) \times \cos(297^{\circ}27'00'')$$

$$= 667.334\text{ft}$$

$$E = 1400.00\text{ft} + (190.00\text{ft} + 173.00\text{ft}) \times \sin(297^{\circ}27'00'')$$

$$= 1077.869\text{ft}$$



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Part A.1 What are the new central angles for the three curves?

Using COGO

From O₂ to O₃:

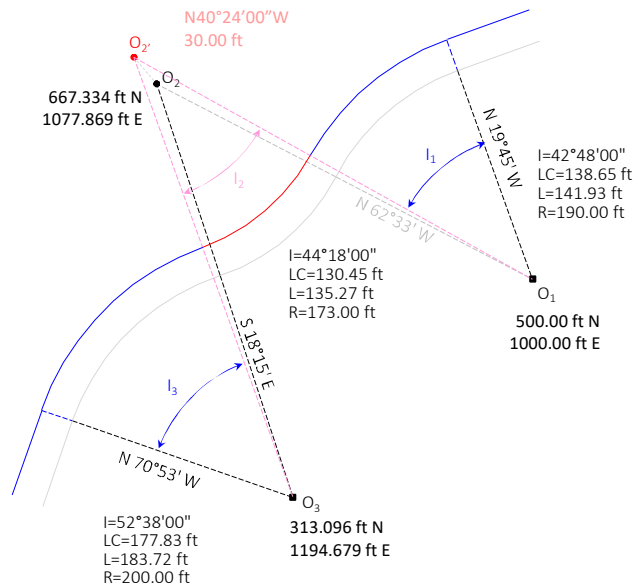
$$Az = 180^{\circ}00'00'' - 18^{\circ}15'00'' = 161^{\circ}45'00''$$

$$N = 667.334\text{ft} + (173.00\text{ft} + 200.00\text{ft}) \times \cos(161^{\circ}45'00'')$$

$$= 313.096\text{ft}$$

$$E = 1077.869\text{ft} + (173.00\text{ft} + 200.00\text{ft}) \times \sin(161^{\circ}45'00'')$$

$$= 1194.679\text{ft}$$



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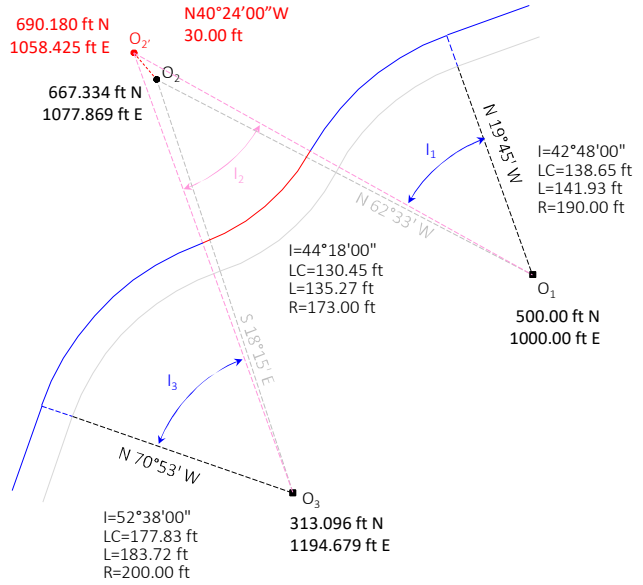
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Part A.1 What are the new central angles for the three curves?

Using COGO

From O_2 to O_2 :

$$\begin{aligned} Az &= 360^\circ 00' 00'' - 40^\circ 24' 00'' = 319^\circ 36' 00'' \\ N &= 667.334\text{ft} + 30.00\text{ft} \times \cos(319^\circ 36' 00'') \\ &= 690.180\text{ft} \\ E &= 1077.869\text{ft} + 30.00\text{ft} \times \sin(319^\circ 36' 00'') \\ &= 1058.425\text{ft} \end{aligned}$$



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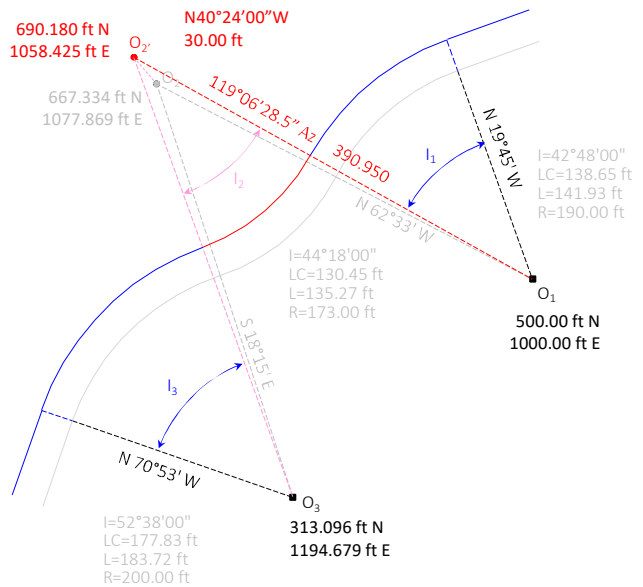
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Part A.1 What are the new central angles for the three curves?

Using COGO

Inverse from O_2 to O_1 :

$$\begin{aligned} \Delta N &= 500.00\text{ft} - 690.180\text{ft} = -190.180\text{ft} \\ \Delta E &= 1400.00\text{ft} - 1058.425\text{ft} = +341.575\text{ft} \\ L &= \sqrt{(-190.180\text{ft})^2 + (+341.575\text{ft})^2} = 390.950\text{ft} \\ \alpha &= \tan^{-1} \left[\frac{+341.575\text{ft}}{-190.180\text{ft}} \right] = -60^\circ 53' 31.5'' \\ Az &= 119^\circ 06' 28.5'' \end{aligned}$$



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Part A.1 What are the new central angles for the three curves?

Using COGO

Inverse from O₂ to O₃:

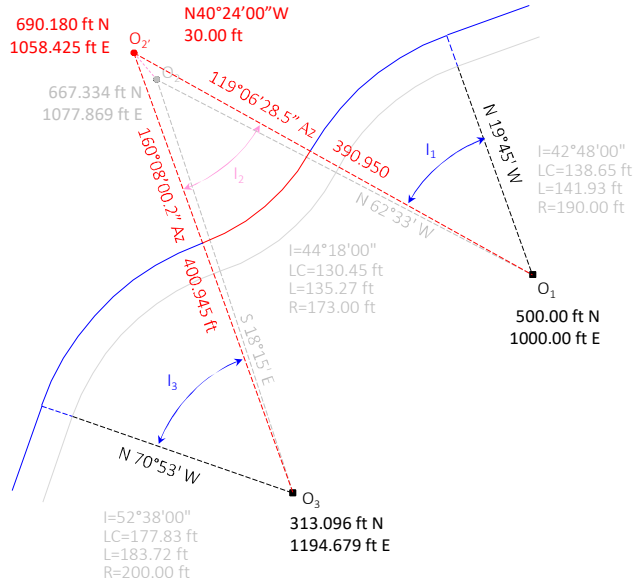
$$\Delta N = 313.096\text{ft} - 690.180\text{ft} = -377.084\text{ft}$$

$$\Delta E = 1194.679\text{ft} - 1058.425\text{ft} = +136.254\text{ft}$$

$$L = \sqrt{(-377.084\text{ft})^2 + (+136.254\text{ft})^2} = 400.945\text{ft}$$

$$\alpha = \tan^{-1} \left[\frac{+136.254\text{ft}}{-377.084\text{ft}} \right] = -19^\circ 51' 59.8''$$

$$Az = 160^\circ 08' 00.2''$$



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Part A.1 What are the new central angles for the three curves?

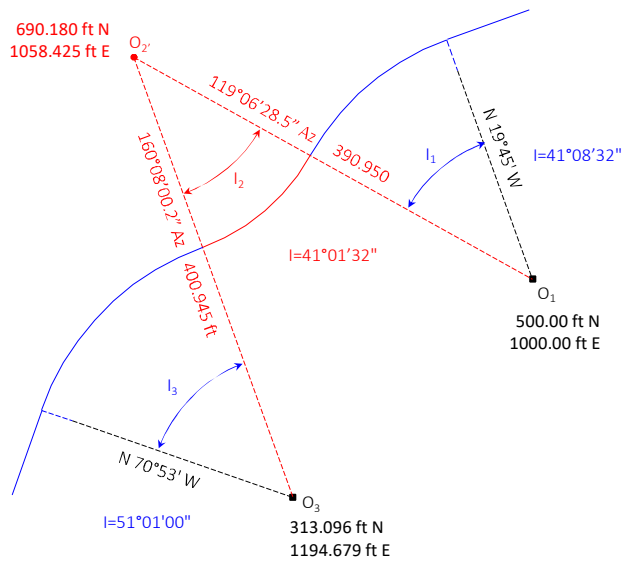
Using COGO

Compute differences between directions:

$$I_1 = (360^\circ 00' 00'' - 19^\circ 45' 00'') - (119^\circ 06' 28.5'' + 180^\circ 00' 00'') = 41^\circ 08' 31.5'' = \underline{41^\circ 08' 32''}$$

$$I_2 = 160^\circ 08' 00.2'' - 119^\circ 06' 28.5'' = 41^\circ 01' 31.7'' = \underline{41^\circ 01' 32''}$$

$$I_3 = (160^\circ 08' 00.2'' + 180^\circ 00' 00'') - (360^\circ 00' 00'' - 70^\circ 53' 00'') = 51^\circ 01' 00.2'' = \underline{51^\circ 01' 00''}$$

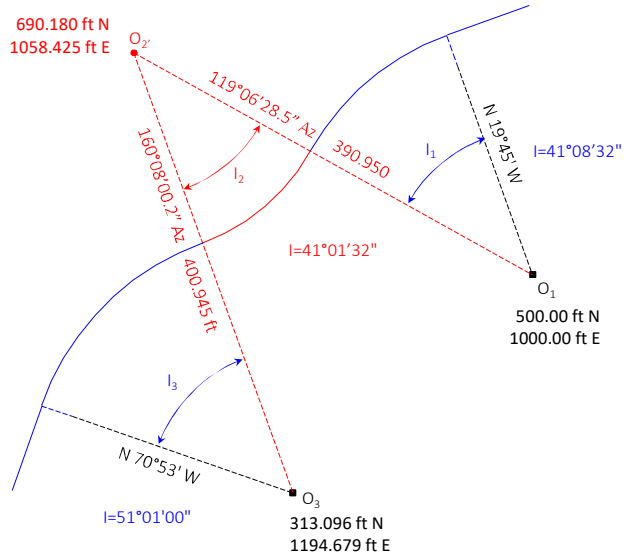


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Part A.1 What are the new central angles for the three curves?

	Curve 1	Curve 2	Curve 3	
a.	41°08'30"	41°01'24"	51°00'54"	Closest...
b.	41°48'08"	42°00'15"	53°38'23"	
c.	42°35'39"	44°18'25"	51°52'02"	
d.	42°40'15"	40°31'00"	52°35'50"	



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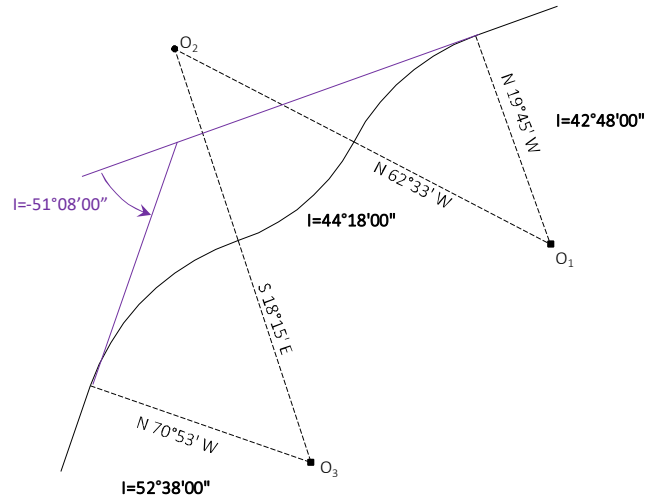
Part A.1 What are the new central angles for the three curves?

Logical analysis?

The total I angle is the sum of the three curves' I angles, right are +, left are -

$$I = (-42^\circ 48' 00'') + 44^\circ 18' 00'' + (-52^\circ 38' 00'')$$

$$= -51^\circ 08' 00''$$



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Part A.1 What are the new central angles for the three curves?

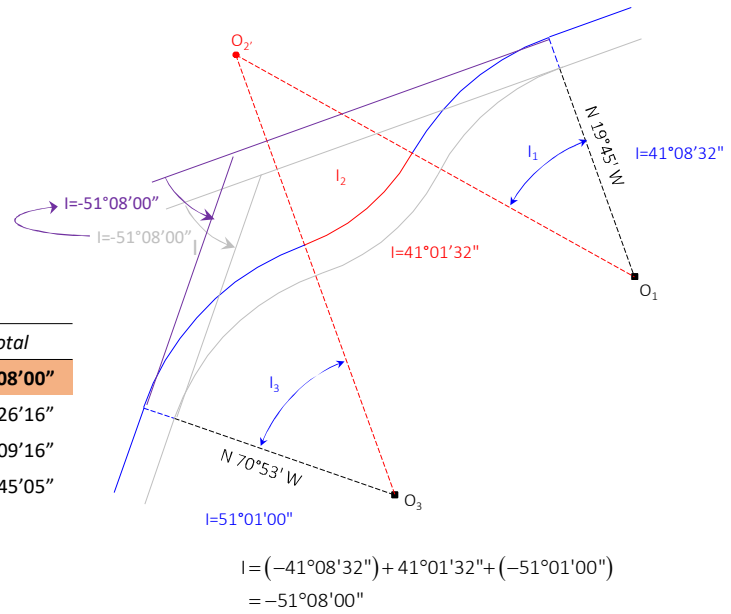
Logical analysis?

This is preserved with the re-alignment.
Compute the totals of the answers.

$$I = (-I_1'') + I_2 + (-I_3) = -51^\circ 08' 00''$$

Only **a** meets the requirement.

	Curve 1 (-)	Curve 2 (+)	Curve 3 (-)	Total
a.	41°08'30"	41°01'24"	51°00'54"	-51°08'00"
b.	41°48'08"	42°00'15"	53°38'23"	-53°26'16"
c.	42°35'39"	44°18'25"	51°52'02"	-50°09'16"
d.	42°40'15"	40°31'00"	52°35'50"	-54°45'05"

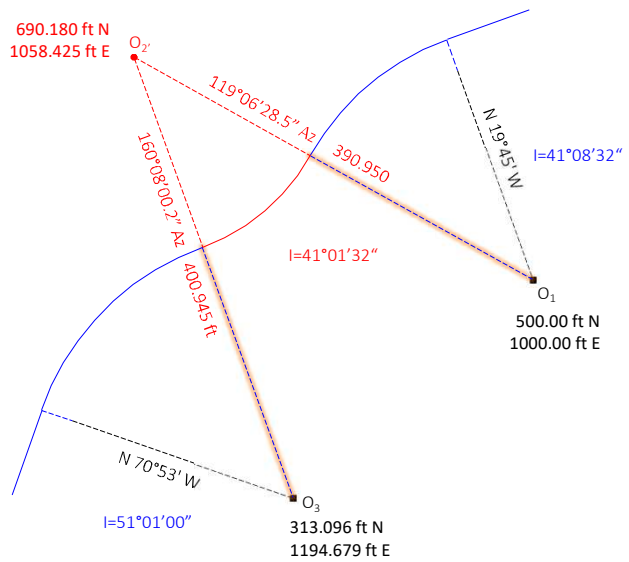


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Part A.2 What are the new new radii of curves 1 and 3?

	Curve 1	Curve 2	Curve 3
a.	190.00 ft	173.00 ft	200.00 ft
b.	190.00 ft	200.93 ft	200.00 ft
c.	204.62 ft	186.31 ft	214.98 ft
d.	217.95 ft	173.00 ft	227.96 ft



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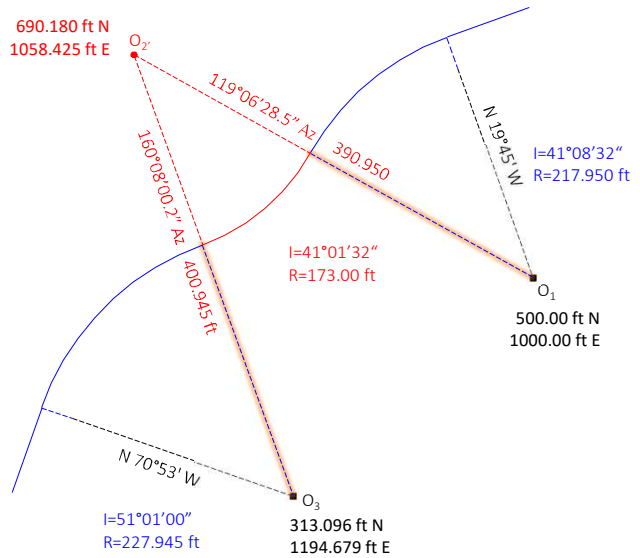
Part A.2 What are the new new radii of curves 1 and 3?

R₂ stays at 173.00 ft. Subtract it from the COGO inverse distances O₁-O₂ and O₂-O₃:

$$R_1 = 390.950\text{ft} - 173.00\text{ft} = 217.950\text{ft}$$

$$R_3 = 400.945\text{ft} - 173.00\text{ft} = 227.945\text{ft}$$

	Curve 1	Curve 2	Curve 3
a.	190.00 ft	173.00 ft	200.00 ft
b.	190.00 ft	200.93 ft	200.00 ft
c.	204.62 ft	186.31 ft	214.98 ft
d.	217.95 ft	173.00 ft	227.96 ft

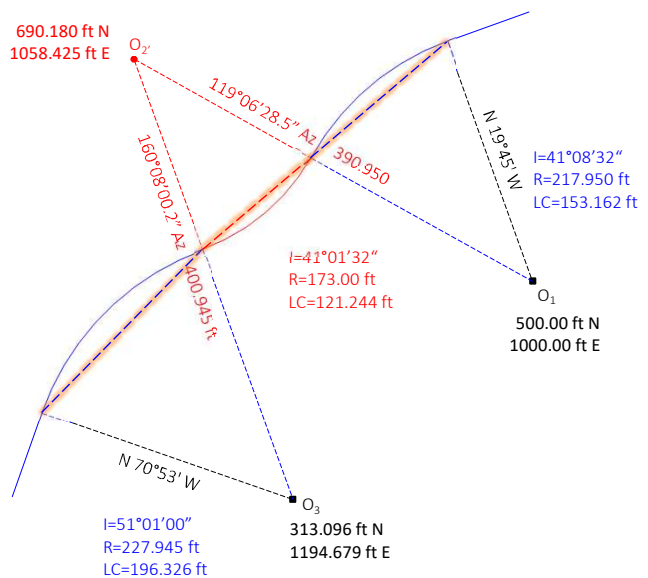


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Part A.3 What are the new long chords of each curve?

	Curve 1	Curve 2	Curve 3
a.	153.16 ft	121.24 ft	196.33 ft
b.	154.66 ft	123.88 ft	202.96 ft
c.	155.32 ft	115.69 ft	202.96 ft
d.	155.48 ft	135.27 ft	135.27 ft



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Part A.3 What are the new long chords of each curve?

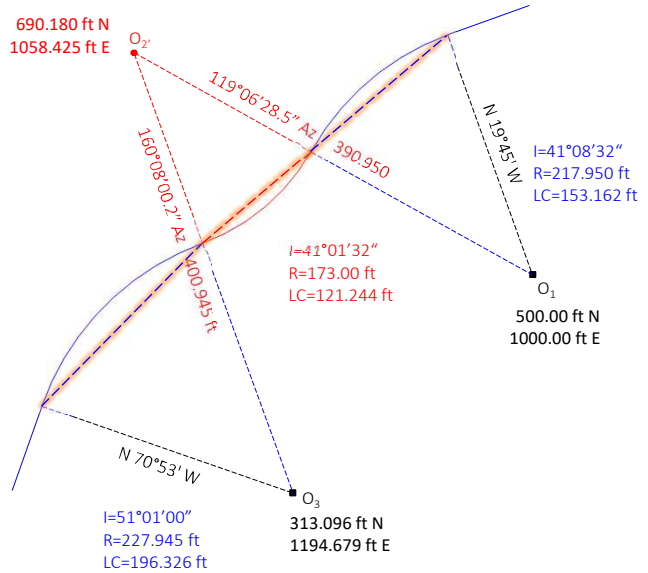
$$LC = 2 \times R \times \sin\left(\frac{I}{2}\right)$$

$$LC_1 = 2 \times 217.95 \text{ ft} \times \sin\left(\frac{41^\circ 08' 32''}{2}\right) = 153.162 \text{ ft}$$

$$LC_2 = 2 \times 173.00 \text{ ft} \times \sin\left(\frac{41^\circ 01' 32''}{2}\right) = 121.244 \text{ ft}$$

$$LC_3 = 2 \times 227.945 \text{ ft} \times \sin\left(\frac{51^\circ 01' 00''}{2}\right) = 196.326 \text{ ft}$$

	Curve 1	Curve 2	Curve 3
a.	153.16 ft	121.24 ft	196.33 ft
b.	154.66 ft	123.88 ft	202.96 ft
c.	155.32 ft	115.69 ft	202.96 ft
d.	155.48 ft	135.27 ft	135.27 ft

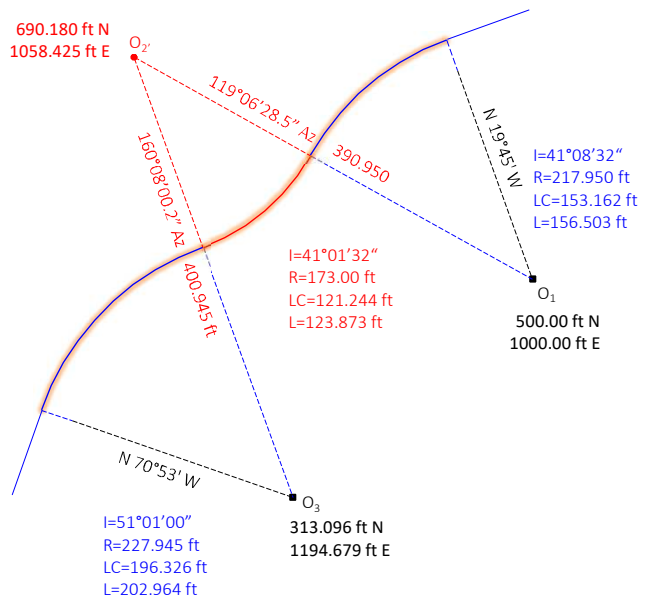


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Part A.4 What are the arc lengths of each curve?

	Curve 1	Curve 2	Curve 3
a.	143.67 ft	127.37 ft	231.07 ft
b.	153.14 ft	121.25 ft	196.33 ft
c.	155.03 ft	128.93 ft	200.86 ft
d.	156.50 ft	123.87 ft	202.97 ft



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Part A.4 What are the arc lengths of each curve?

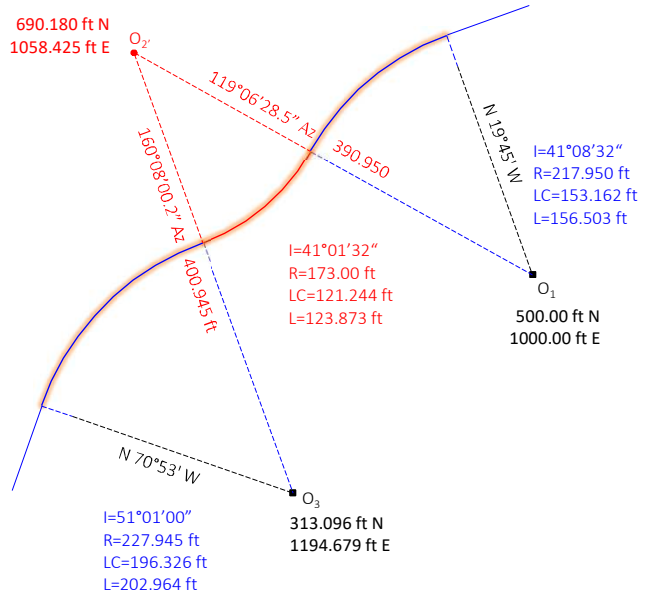
$$L = R \times I_r = R \times I \times \left(\frac{\pi}{180^\circ}\right)$$

$$L_1 = 217.950 \text{ ft} \times 41^\circ 08' 32'' \times \left(\frac{\pi}{180^\circ}\right) = 156.503 \text{ ft}$$

$$L_2 = 173.00 \text{ ft} \times 41^\circ 01' 32'' \times \left(\frac{\pi}{180^\circ}\right) = 123.873 \text{ ft}$$

$$L_3 = 227.945 \text{ ft} \times 51^\circ 01' 00'' \times \left(\frac{\pi}{180^\circ}\right) = 202.964 \text{ ft}$$

Curve 1	Curve 2	Curve 3
a. 143.67 ft	127.37 ft	231.07 ft
b. 153.14 ft	121.25 ft	196.33 ft
c. 155.03 ft	128.93 ft	200.86 ft
d. 156.50 ft	123.87 ft	202.97 ft

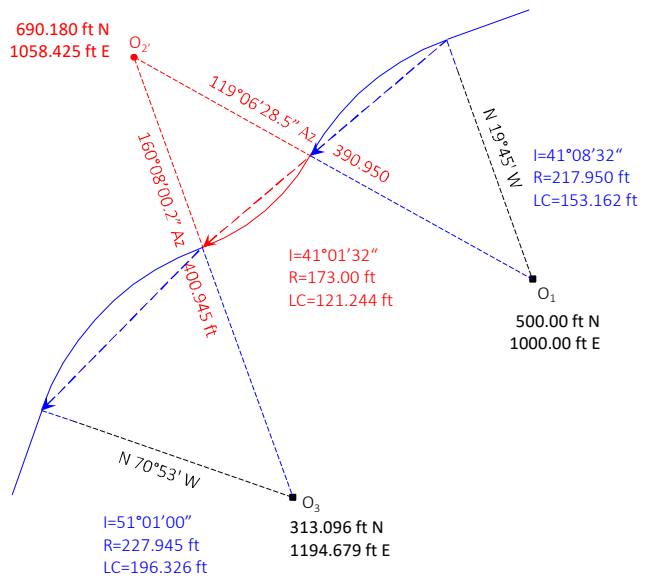


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Part A.5 What is the bearing of each long chord?

Curve 1	Curve 2	Curve 3
a. S48°51'00"W	S49°36'00"W	S45°26'00"W
b. S49°15'54"W	S49°36'51"W	S45°01'48"W
c. S49°40'45"W	S49°37'18"W	S44°37'44"W
d. S49°51'25"W	S49°26'09"W	S44°59'43"W



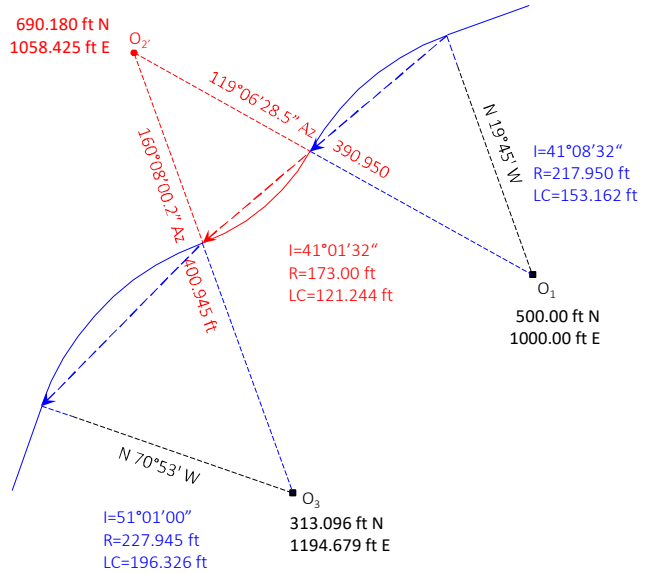
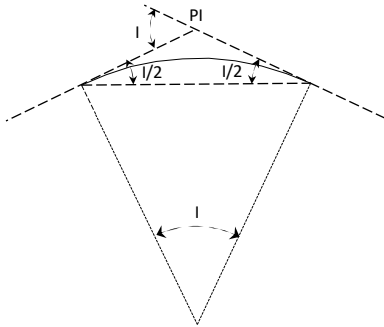
Mentoring Mondays
Math Problem Examples

Jerry Mahun 01 Nov 2021

Part A.5 What is the bearing of each long chord?

For a circular curve:

The deflection angle between tangents at the PI is I.
The angle between the chord and the tangent at either end of the curve is (I/2)



Mentoring Mondays
Math Problem Examples

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Part A.5 What is the bearing of each long chord?

Azimuth of the first tangent PC₁-PI₁:

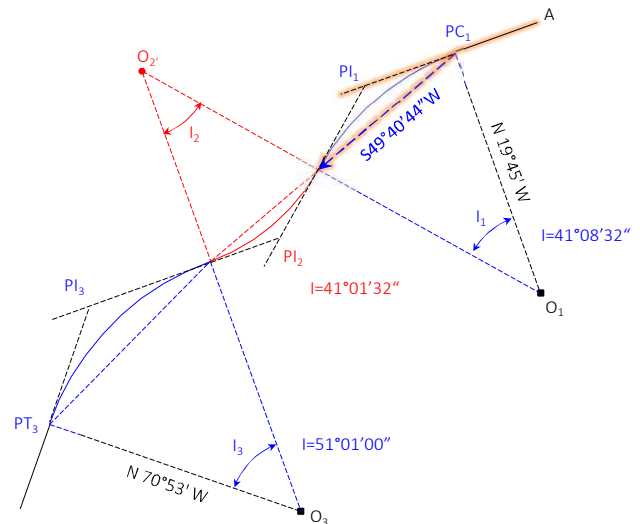
$$Az = (180°00'00'' - 19°45') + 90°00'00'' = 250°15'00''$$

Azimuth of Curve 1 chord

$$Az = 250°15'00'' + \left(\frac{-41°08'32''}{2} \right) = 229°40'44''$$

Bearing of Curve 1 chord

$$\alpha = 229°40'44'' - 180°00'00'' = 49°40'44'' \Rightarrow \underline{S49°40'44''W}$$





Mentoring Mondays Math Problem Examples

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Part A.5 What is the bearing of each long chord?

Azimuth of the target PI_1-PI_2 :

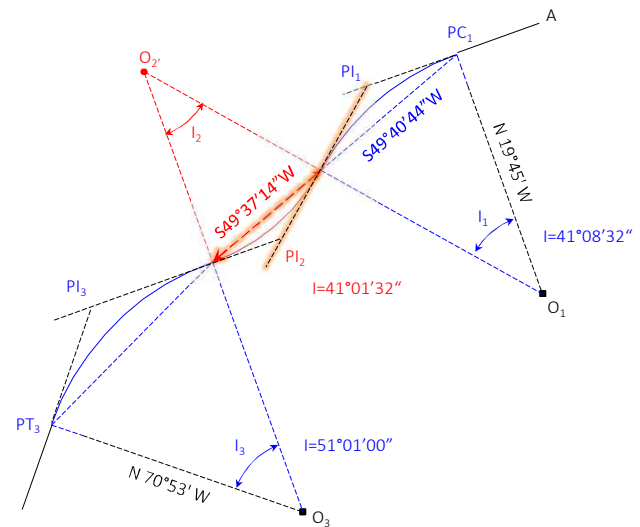
$$\begin{aligned} Az &= 250^\circ 15' 00'' + (-41^\circ 08' 32'') \\ &= 209^\circ 06' 28'' \end{aligned}$$

Azimuth of Curve 2 chord

$$\begin{aligned} Az &= 209^\circ 06' 28'' + \left(\frac{41^\circ 01' 32''}{2} \right) \\ &= 229^\circ 37' 14'' \end{aligned}$$

Bearing of Curve 2 chord

$$\begin{aligned} \beta &= 229^\circ 37' 14'' - 180^\circ 00' 00'' \\ &= 49^\circ 37' 14'' \\ &\Rightarrow \underline{S49^\circ 37' 14'' W} \end{aligned}$$



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Part A.5 What is the bearing of each long chord?

Azimuth of the target PI_2-PI_3 :

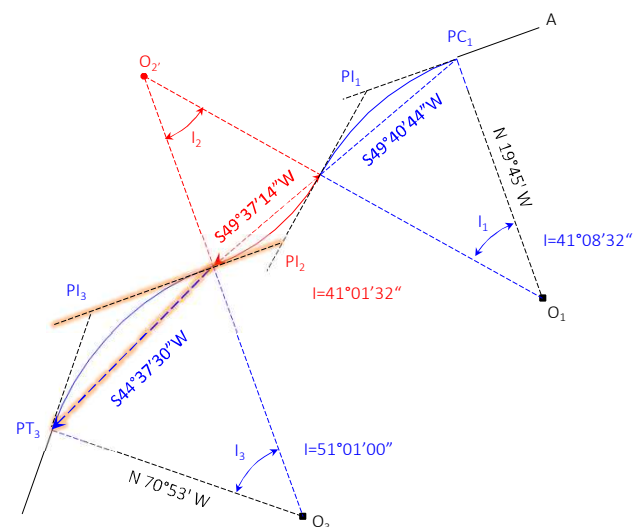
$$\begin{aligned} Az &= 209^\circ 06' 28'' + 41^\circ 01' 32'' \\ &= 250^\circ 08' 00'' \end{aligned}$$

Azimuth of Curve 3 chord

$$\begin{aligned} Az &= 250^\circ 08' 00'' + \left(\frac{-51^\circ 01' 00''}{2} \right) \\ &= 224^\circ 37' 30'' \end{aligned}$$

Bearing of Curve 3 chord

$$\begin{aligned} \beta &= 224^\circ 37' 30'' - 180^\circ 00' 00'' \\ &= 44^\circ 37' 30'' \\ &\Rightarrow \underline{S44^\circ 37' 30'' W} \end{aligned}$$



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Math Problem Examples

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Part A.5 What is the bearing of each long chord?

Math check: Azimuth of the tangent PI_3-PT_3 :

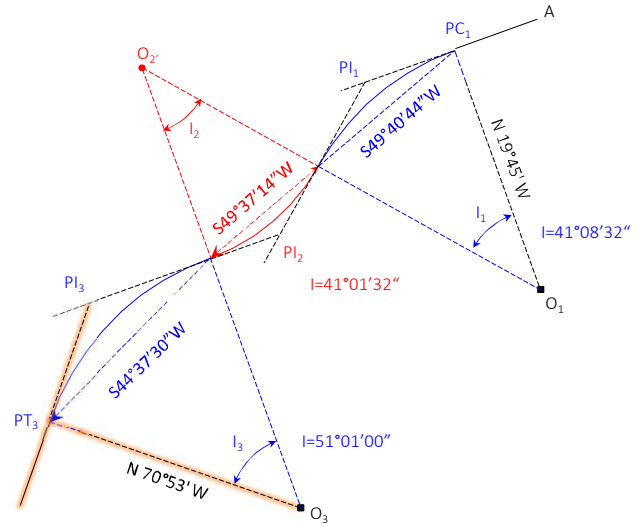
$$Az = 250^{\circ}08'00'' + (-51^{\circ}01'00'') \\ = 199^{\circ}07'00''$$

Azimuth of Curve 3 radius at PT_3 :

$$Az = 199^{\circ}07'00'' - 90^{\circ}00'00'' \\ = 109^{\circ}07'00''$$

Bearing of Curve 3 radius at PT_3 :

$$\beta = 180^{\circ}00'00'' - 109^{\circ}07'00'' \\ = 70^{\circ}53'00'' \\ \Rightarrow S70^{\circ}53'00''E \text{ check}$$



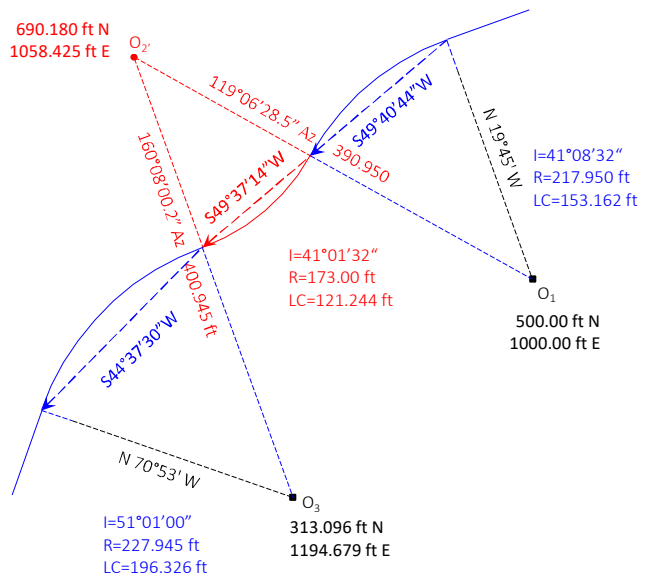
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Math Problem Examples

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Part A.5 What is the bearing of each long chord?

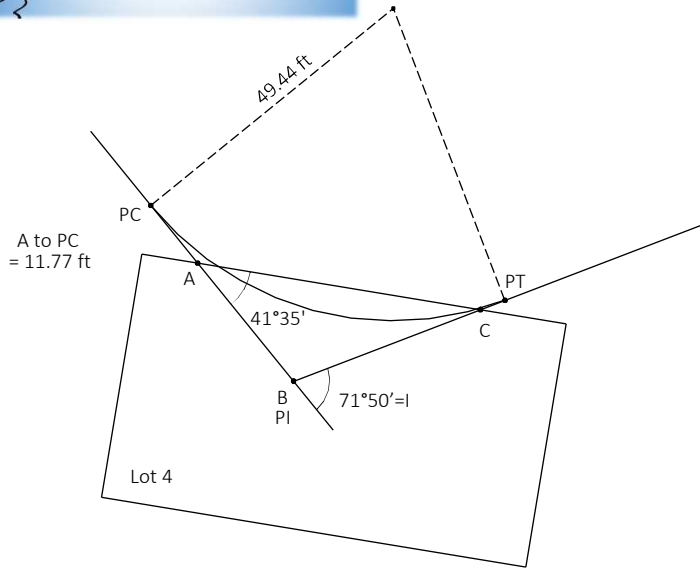
	Curve 1	Curve 2	Curve 3
a.	S48°51'00"W	S49°36'00"W	S45°26'00"W
b.	S49°15'54"W	S49°36'51"W	S45°01'48"W
c.	S49°40'45"W	S49°37'18"W	S44°37'44"W
d.	S49°51'25"W	S49°26'09"W	S44°59'43"W

Closest...



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Math Problem Examples

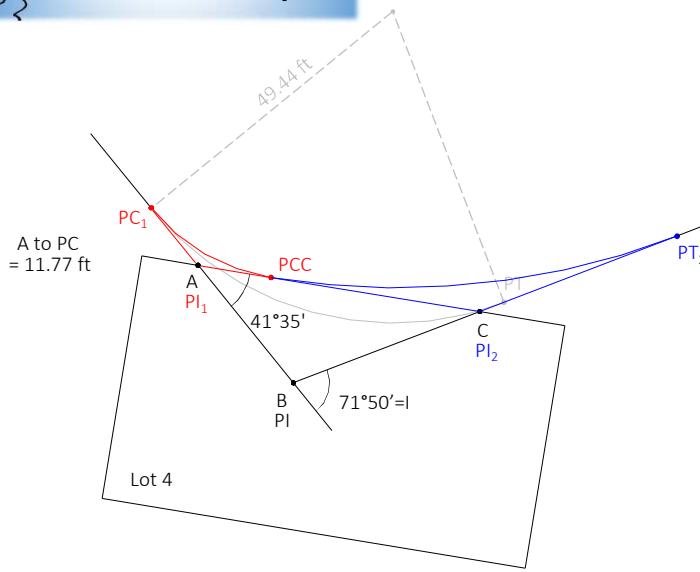
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Problem B. The circular curve encroaches onto Lot 4. It will be replaced by a compound curve tangent to the northern boundary of Lot 4.

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Math Problem Examples

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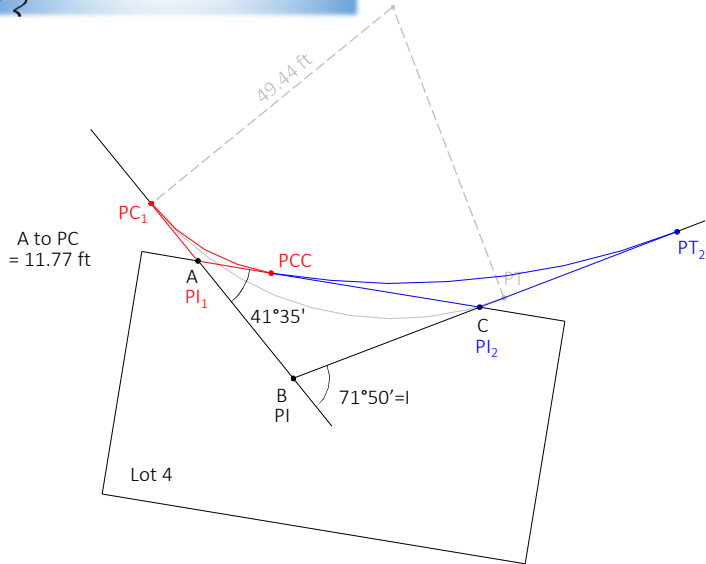


Problem B. The circular curve encroaches onto Lot 4. It will be replaced by a compound curve tangent to the northern boundary of Lot 4.

The existing PC will be the start of the first curve and point A will serve as its PI, C will be the PI of the second curve.

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Math Problem Examples

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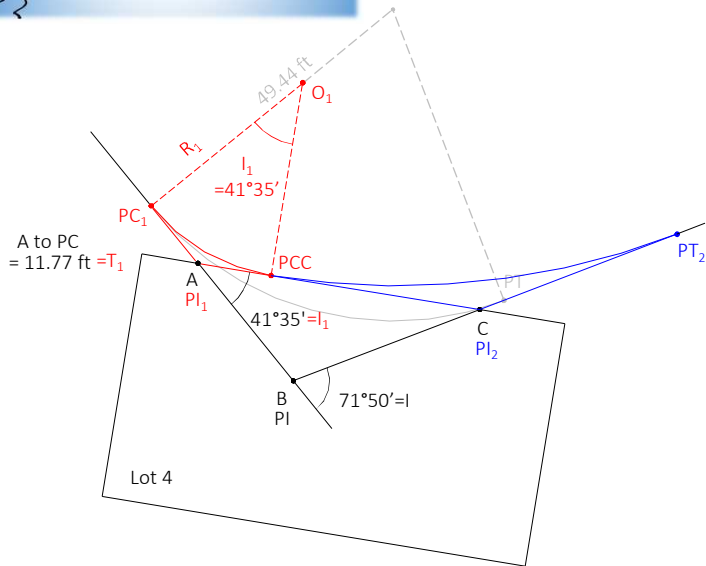


Part B.1 What is the radius of the first curve?

- a. 31.00 ft
- b. 47.35 ft
- c. 49.44 ft
- d. 52.33 ft

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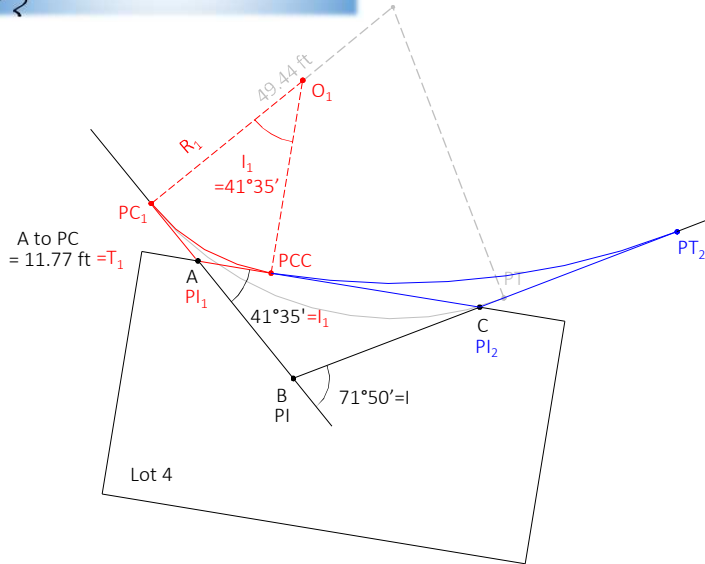


Part B.1 What is the radius of the first curve?

Label the curve parts we know.

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Math Problem Examples

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Part B.1 What is the radius of the first curve?

Label the curve parts we know.

Compute the radius from the tangent distance.

$$T_1 = R_1 \times \tan\left(\frac{I_1}{2}\right)$$

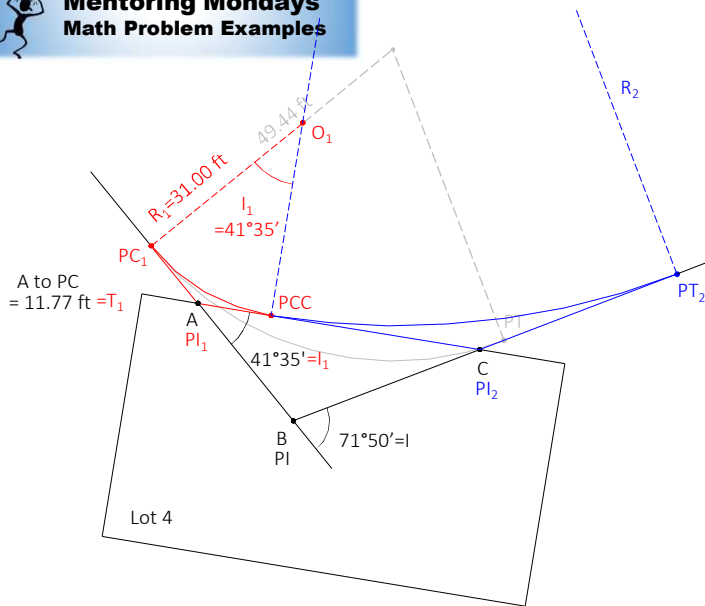
$$R_1 = \frac{T_1}{\tan\left(\frac{I_1}{2}\right)} = \frac{11.77\text{ft}}{\tan\left(\frac{41^\circ35'}{2}\right)}$$

$$= 30.998\text{ft} = \underline{31.00\text{ft}}$$

Answer: a. 31.00 ft

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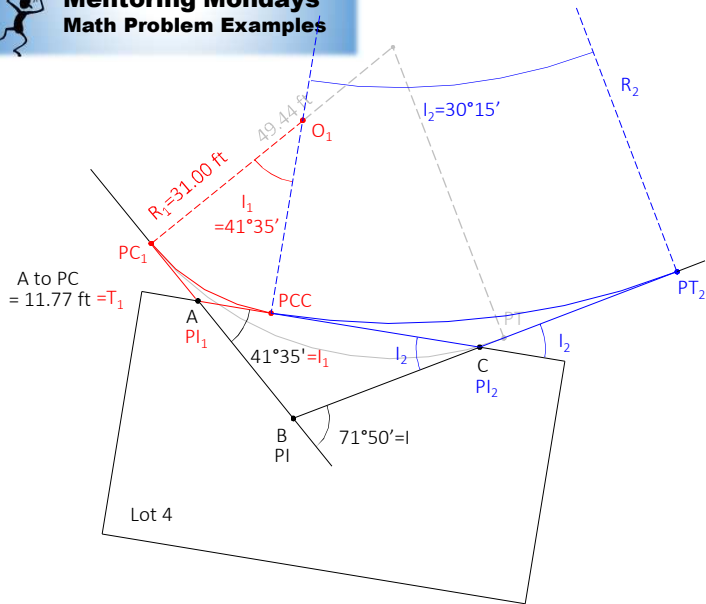


Part B.2 What is the radius of the second curve?

- a. 121.35 ft
- b. 122.30 ft
- c. 123.25 ft
- d. 124.20 ft

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Part B.2 What is the radius of the second curve?

We need at least two parts of a curve to determine the remainder of its attributes.

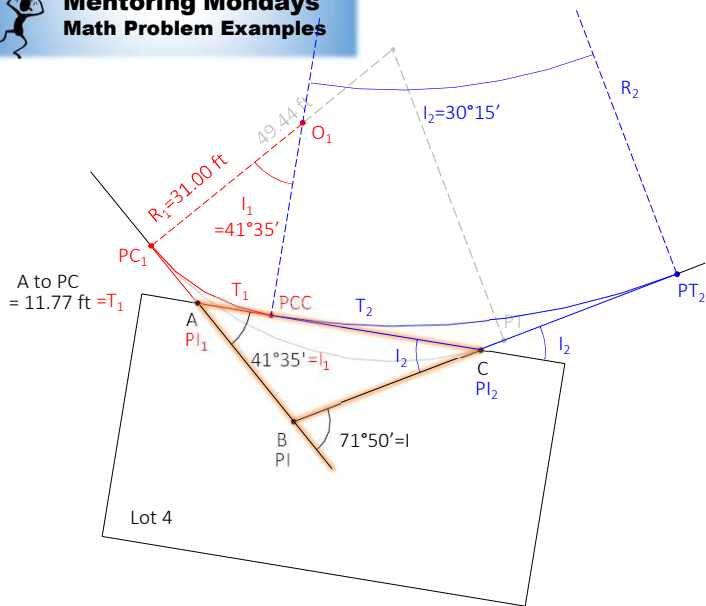
We can determine the I-angle for the second curve:

$$I_2 = I - I_1 = 71^\circ 50' - 41^\circ 35' = 30^\circ 15'$$

We still need another piece.

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Part B.1 What is the radius of the first curve?

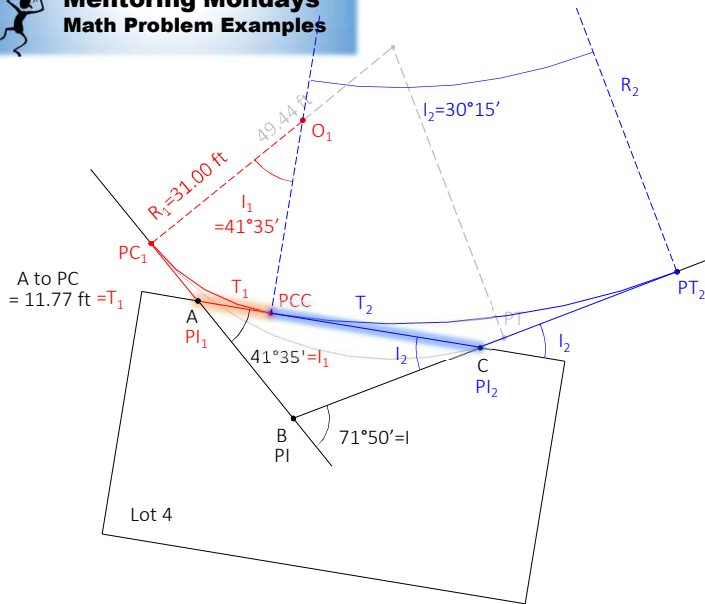
The three PIs define the *vertex triangle*.

Angle at PI is $(180^\circ - I)$

Distance from PI_1 to PCC is T_1
Distance from PCC to PI_2 is T_2

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Part B.2 What is the radius of the second curve?

That piece is the tangent distance, T_2 which is $(AC - T_1)$.

How to get AC?

In triangle ABC, we start with three angles:

$$I_1, I_2, 180^\circ - (I_1 + I_2)$$

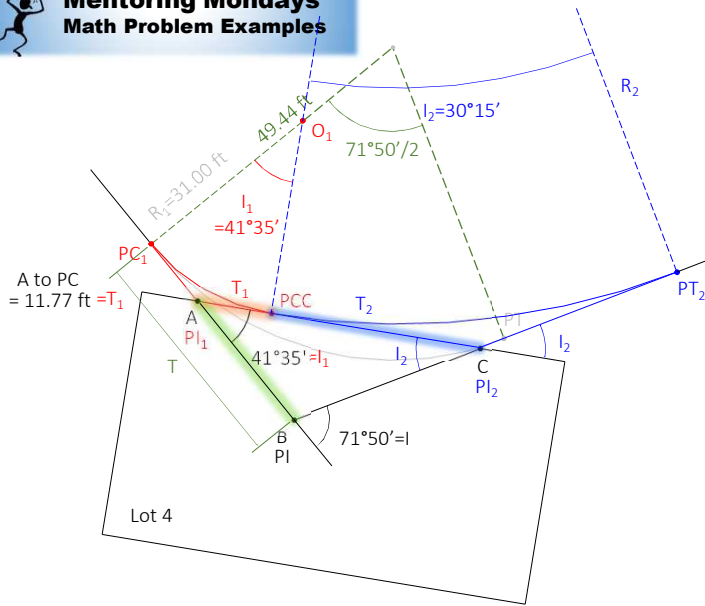
To solve AC, use Law of Sines:

$$\frac{AC}{\sin(180^\circ - I_1 - I_2)} = \frac{AB}{\sin(I_2)} = \frac{BC}{\sin(I_1)}$$

We don't have any sides ... or do we ...?

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Math Problem Examples

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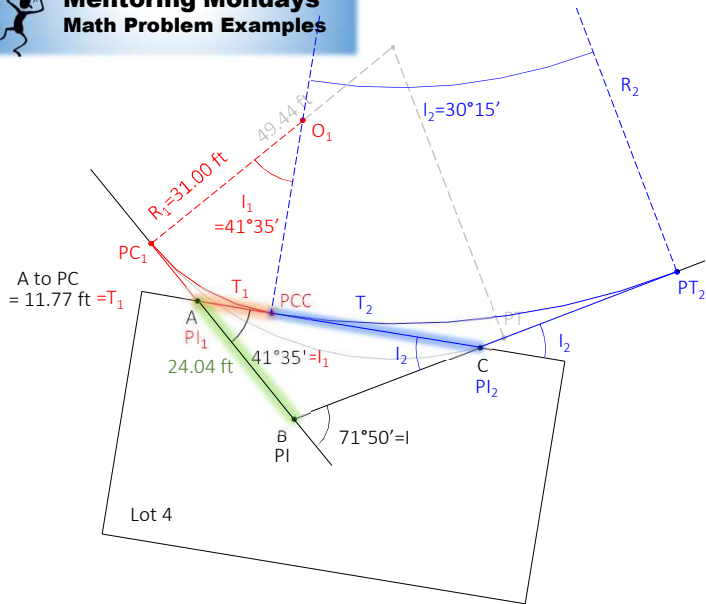
$$\frac{AC}{\sin(180^\circ - I_1 - I_2)} = \frac{AB}{\sin(I_2)} = \frac{BC}{\sin(I_1)}$$

We don't have any sides ... or do we ...?

$$\begin{aligned} AB &= T - T_1 \\ &= 49.44 \text{ ft} \times \tan\left(\frac{71^\circ 50'}{2}\right) - 11.77 \text{ ft} \\ &= 35.81 \text{ ft} - 11.77 \text{ ft} = 24.04 \text{ ft} \end{aligned}$$

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Part B.2 What is the radius of the second curve?

That piece is the tangent distance, T_2 which is $(AC - T_1)$.

How to get AC?
In triangle ABC, we start with three angles:

$$I_1, I_2, 180^\circ - (I_1 + I_2)$$

To solve AC, use Law of Sines:

$$\frac{AC}{\sin(180^\circ - 41^\circ 35' - 30^\circ 15')} = \frac{24.04 \text{ ft}}{\sin(30^\circ 15')}$$

$$AC = 24.04 \text{ ft} \times \frac{\sin(108^\circ 10')}{\sin(30^\circ 15')} = 45.34 \text{ ft}$$

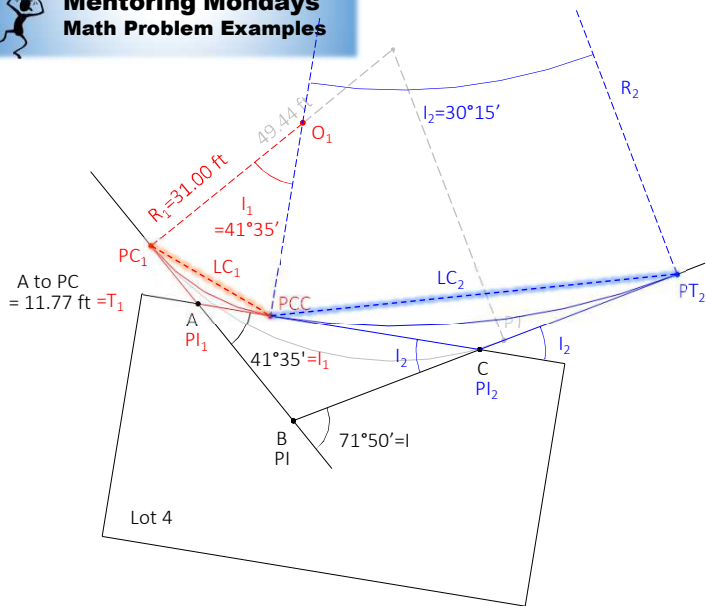
$$\text{Then: } T_2 = (45.34 \text{ ft} - 11.77 \text{ ft}) = 33.57$$

$$\text{So: } R_2 = \frac{T_2}{\tan\left(\frac{I_2}{2}\right)} = \frac{33.57}{\tan\left(\frac{30^\circ 15'}{2}\right)} = 124.20 \text{ ft}$$

Answer: d. 124.20 ft

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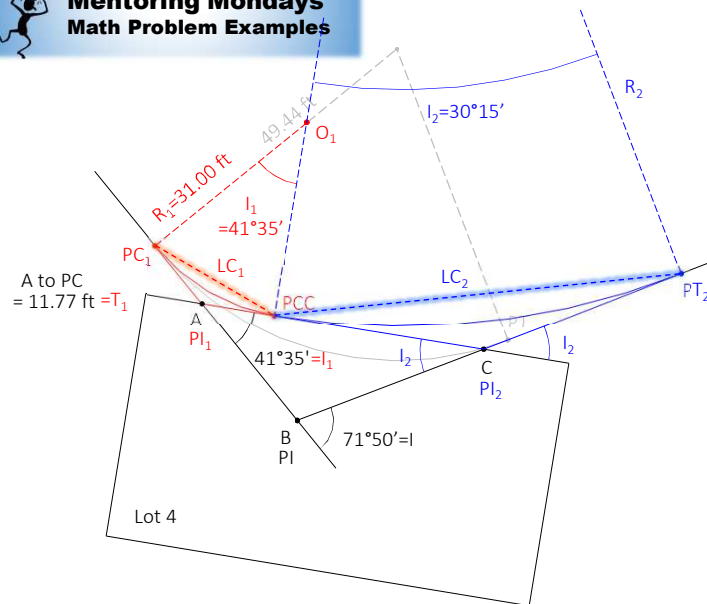
Part B.3 What are the long chords for both curves?

- a. 20.18 ft and 50.43 ft
- b. 20.18 ft and 64.21 ft
- c. 22.01 ft and 64.81 ft
- d. 23.89 ft and 64.81 ft



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Part B.3 What are the long chords for both curves?

$$LC = 2R \times \sin\left(\frac{I}{2}\right)$$

$$LC_1 = 2(31.00\text{ft}) \times \sin\left(\frac{41^\circ 35'}{2}\right) = 22.01\text{ft}$$

$$LC_2 = 2(124.20\text{ft}) \times \sin\left(\frac{30^\circ 15'}{2}\right) = 64.81\text{ft}$$

Answer: c. 22.01 ft and 64.81 ft



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Problem C. Total station on lookout tower, inst ht 5.2 ft. $115^\circ 22' 06''$ and $129^\circ 15' 49''$ zenith angles to far & near sides of a river. Later, horiz dist between two river points meas'd as 225.35 ft. River elev: 841.2 ft. Tower elev?

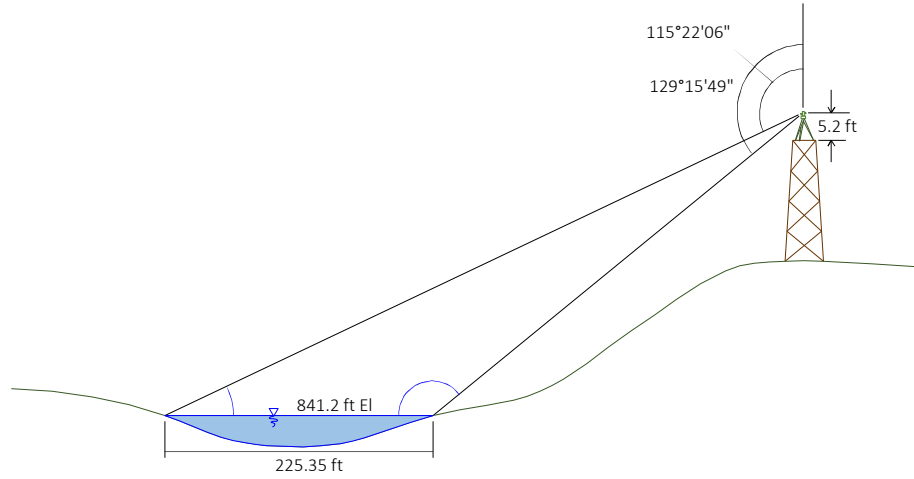
- 954.4 ft
- 1090.4 ft
- 1095.6 ft
- 1243.2 ft

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Math Problem Examples

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Problem C. Total station on lookout tower, inst ht 5.2 ft. $115^{\circ}22'06''$ and $129^{\circ}15'49''$ zenith angles to far & near sides of a river. Later, horiz dist between two river points meas'd as 225.35 ft. River elev: 841.2 ft. Tower elev?

Sketch



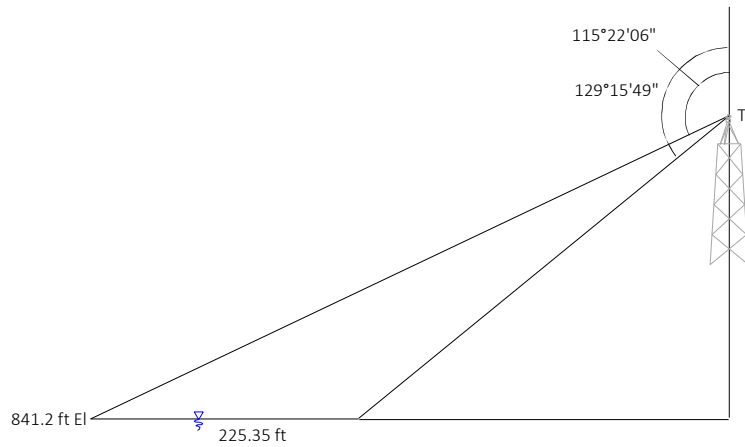
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Math Problem Examples

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Sketch

Isolate triangles



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Math Problem Examples

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Comps

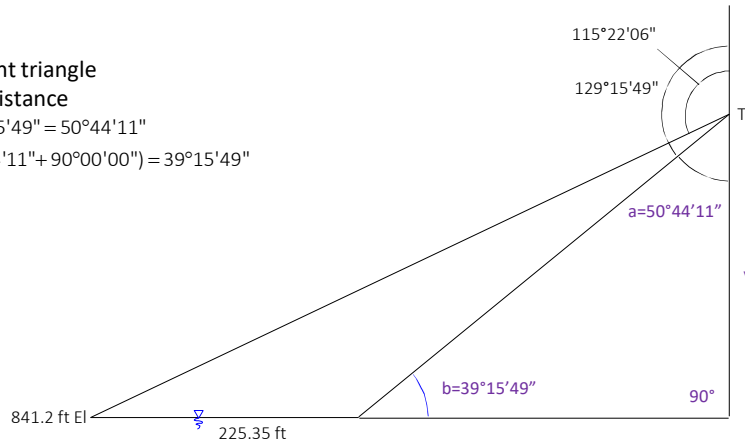
Isolate triangles

Compute angles in right triangle

V is the vertical distance

$$a = 180^{\circ}00'00'' - 129^{\circ}15'49'' = 50^{\circ}44'11''$$

$$b = 180^{\circ}00'00'' - (50^{\circ}44'11'' + 90^{\circ}00'00'') = 39^{\circ}15'49''$$



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Comps

Isolate triangles

Compute angles in right triangle

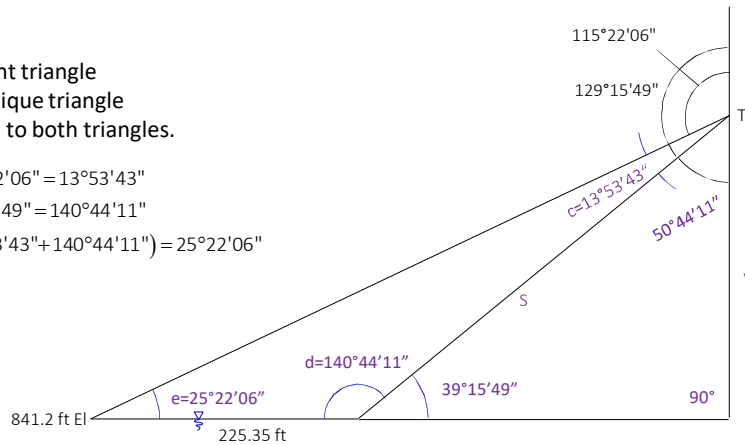
Compute angles in oblique triangle

Side S is common to both triangles.

$$c = 129^{\circ}15'49'' - 115^{\circ}22'06'' = 13^{\circ}53'43''$$

$$d = 180^{\circ}00'00'' - 39^{\circ}15'49'' = 140^{\circ}44'11''$$

$$e = 180^{\circ}00'00'' - (13^{\circ}53'43'' + 140^{\circ}44'11'') = 25^{\circ}22'06''$$





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Problem C. Total station on lookout tower, inst ht 5.2 ft. $115^{\circ}22'06''$ and $129^{\circ}15'49''$ zenith angles to far & near sides of a river. Later, horiz dist between two river points meas'd as 225.35 ft. River elev: 841.2 ft. Tower elev?

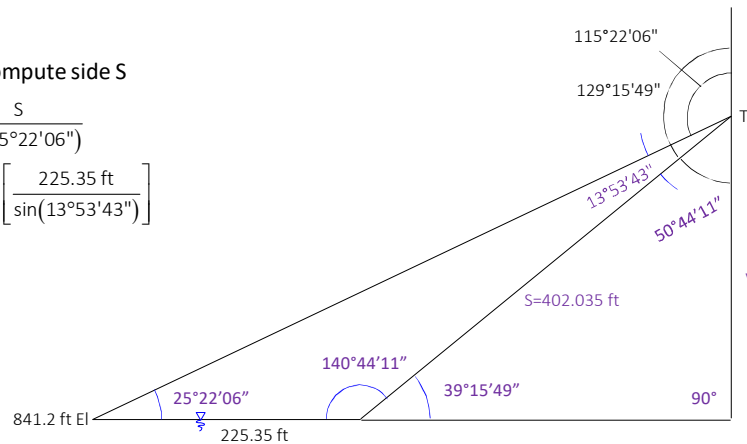
Comps

Use Law of Sines to compute side S

$$\frac{225.35 \text{ ft}}{\sin(13^{\circ}53'43'')} = \frac{S}{\sin(25^{\circ}22'06'')}$$

$$\Rightarrow S = \sin(25^{\circ}22'06'') \times \left[\frac{225.35 \text{ ft}}{\sin(13^{\circ}53'43'')} \right]$$

$$= 402.035 \text{ ft}$$



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Problem C. Total station on lookout tower, inst ht 5.2 ft. $115^{\circ}22'06''$ and $129^{\circ}15'49''$ zenith angles to far & near sides of a river. Later, horiz dist between two river points meas'd as 225.35 ft. River elev: 841.2 ft. Tower elev?

Comps

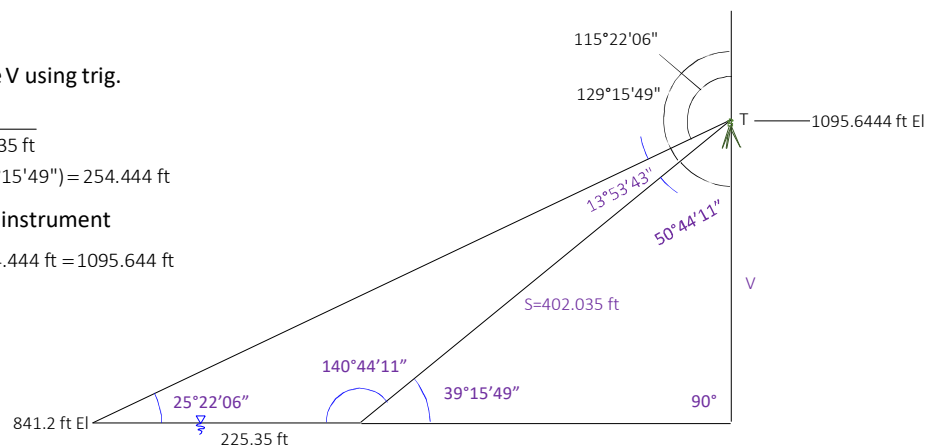
Solve vertical distance V using trig.

$$\sin(39^{\circ}15'49'') = \frac{V}{402.035 \text{ ft}}$$

$$V = 402.035 \text{ ft} \times \sin(39^{\circ}15'49'') = 254.444 \text{ ft}$$

Compute elevation of instrument

$$\text{Elev}_T = 841.2 \text{ ft} + 254.444 \text{ ft} = 1095.644 \text{ ft}$$



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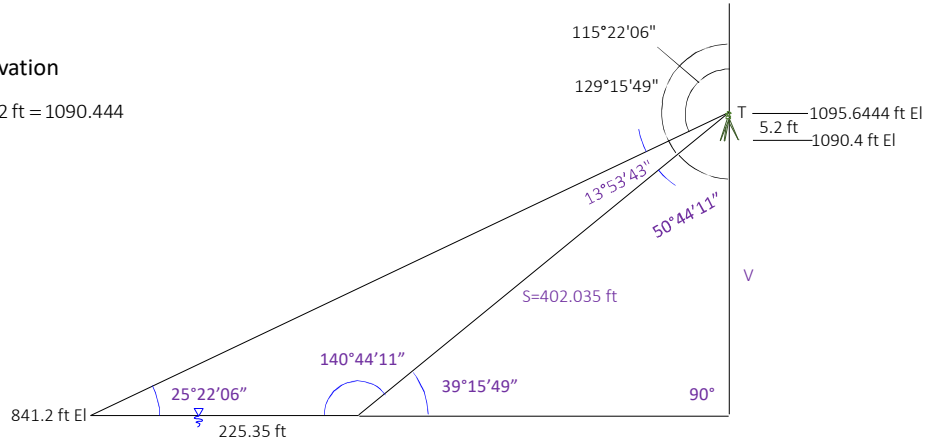
Problem C. Total station on lookout tower, inst ht 5.2 ft. $115^{\circ}22'06''$ and $129^{\circ}15'49''$ zenith angles to far & near sides of a river. Later, horiz dist between two river points meas'd as 225.35 ft. River elev: 841.2 ft. Tower elev?

Comps

Compute platform elevation

$$\begin{aligned} \text{Elev}_p &= 1095.644 \text{ ft} - 5.2 \text{ ft} = 1090.444 \\ &= \underline{1090.4 \text{ ft}} \end{aligned}$$

Answer: b. 1090.4 ft

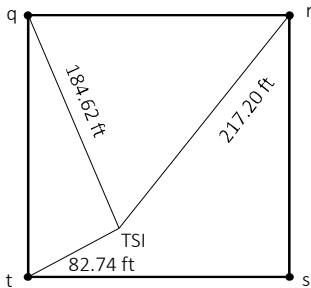


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Math Problem Examples

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Problem D. A surveyor set up her total station instrument (TSI) at location from which she could see three corners of a square parcel. She measured horizontal distance to the three corners as shown below. What is the parcel's area to the nearest 10 square feet?

- a. 15,520 sq ft
- b. 28,060 sq ft
- c. 35,450 sq ft
- d. 43,530 sq ft

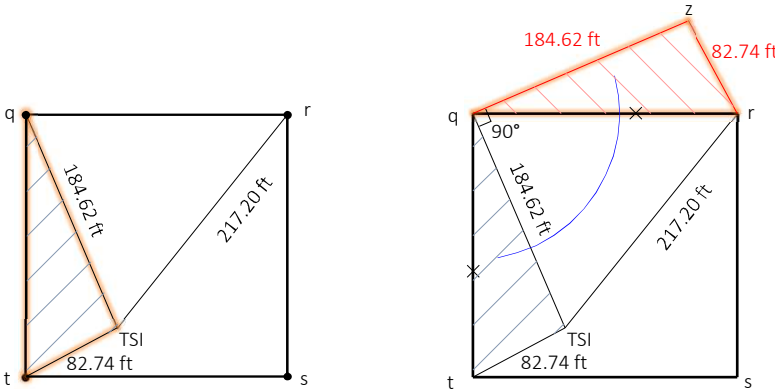


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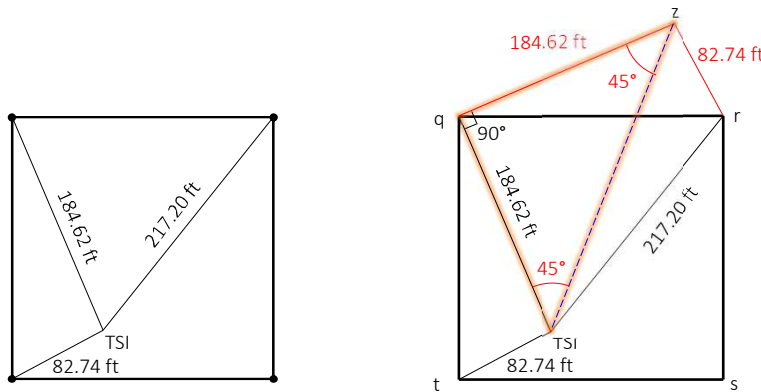
Rotate a copy of triangle q-t-TSI ccw about q so q-t coincides with q-r. This forms a similar triangle q-r-z

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- a. 15,520 sq ft
- b. 28,060 sq ft
- c. 35,450 sq ft
- d. 43,530 sq ft



Triangle q-z-TSI is a right isosceles triangle: angle q is 90°, angles z and TSI are 45°00'00".

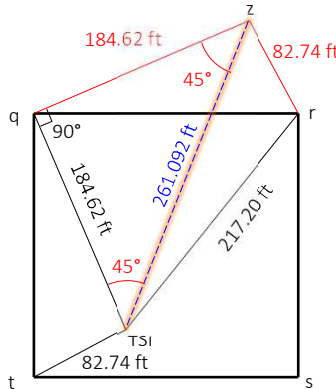
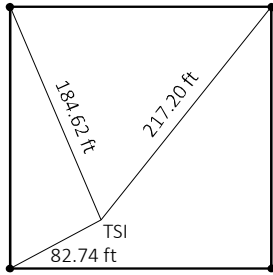


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- c. 35,450 sq ft
- d. 43,530 sq ft



Triangle q-z-TSI is a right isosceles triangle: angle q is 90° , angles z and TSI are $45^\circ 00' 00''$.

Solve side TSI-z using pythagorean theorem: **261.092 ft**

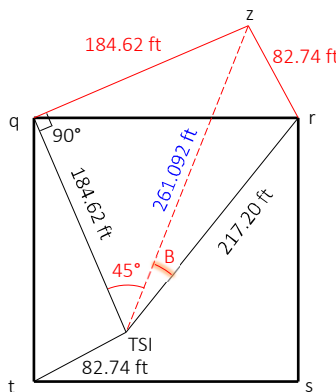
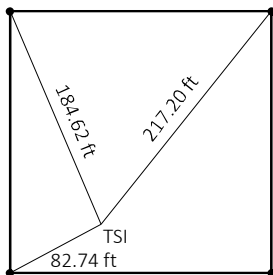


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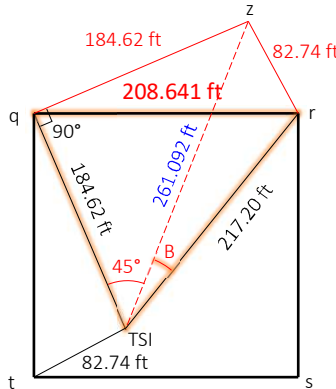
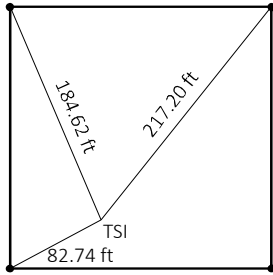
Solve angle B (at TSI) of triangle z-r-TSI using Law of Cosines: **$16^\circ 56' 12.9''$**

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- c. 35,450 sq ft
- d. 43,530 sq ft



Solve q-r of triangle q-r-TSI using Law of Cosines;

The angle at the TSI is
 $A+B = 45^{\circ}00'00'' + 16^{\circ}56'12.9''$
 $= 61^{\circ}56'12.9''$

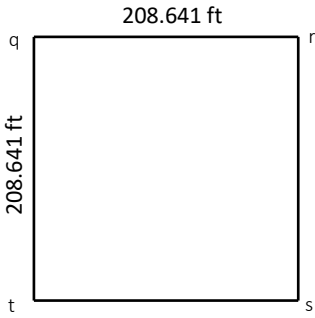
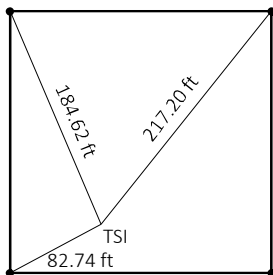
q-r = 208.641 ft

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Math Problem Examples

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Problem D. A surveyor set up her total station instrument (TSI) at location from which she could see three corners of a square parcel. She measured horizontal distance to the three corners as shown below. What is the parcel's area to the nearest 10 square feet?

- a. 15,520 sq ft
- b. 28,060 sq ft
- c. 35,450 sq ft
- d. 43,530 sq ft**



Since the parcel is square,
 Area = 208.641 ft x 208.641
 $= 43,531.1$
 $= 43,530$ sf



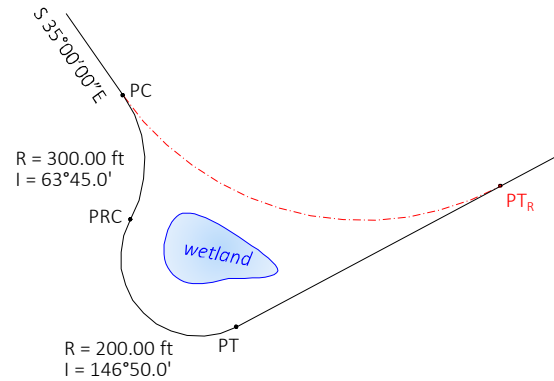
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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

Part E.1. What is the radius, to the nearest 0.1 ft, of the replacement single curve?

- 500.0 ft
- 653.2 ft
- 744.3 ft
- 895.2 ft



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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

Initial Comps

The I angle is the direction change from the incoming tangent to outgoing tangent

For the existing curves:

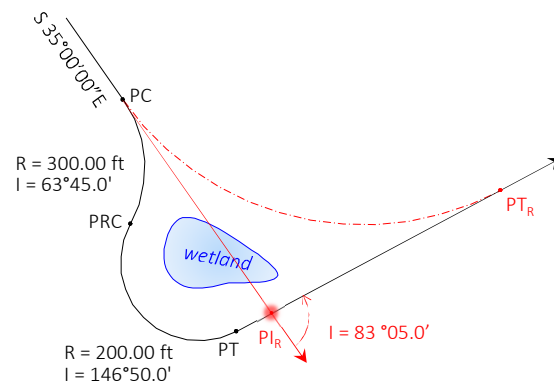
$$63^\circ45.0'R + 146^\circ50.0'L = 83^\circ05.0'L = I$$

Need another curve attribute to fix its geometry.

Compute PI_R coordinates by COGO intersection

Inverse $PC-PI_R$ to get tangent distance, R

Use I and T to compute R



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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

Azimuths:

$$AZ_{PC-PI_1} = 180^\circ 00' 00'' - 35^\circ 00' 00'' = 145^\circ 00' 00''$$

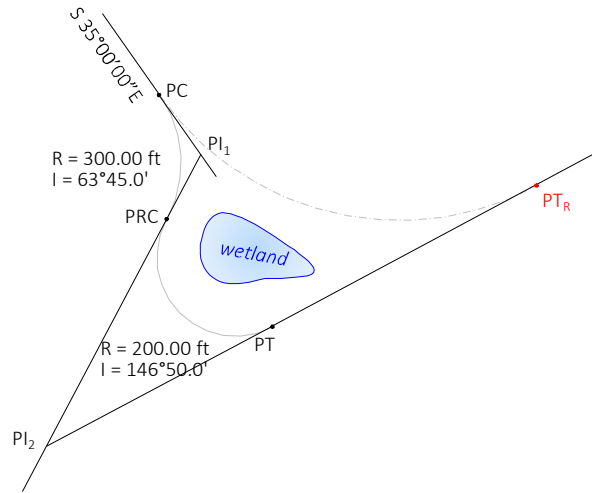
$$AZ_{PI_1-PI_2} = 145^\circ 00' 00'' + 63^\circ 45.0' = 208^\circ 45.0'$$

$$AZ_{PI_2-PT_2} = 208^\circ 45.0' - 146^\circ 50.0' = 61^\circ 55.0'$$

Distances:

$$T_1 = 300.00 \text{ ft} \times \tan\left(\frac{63^\circ 45.0'}{2}\right) = 186.552 \text{ ft}$$

$$T_2 = 200.00 \text{ ft} \times \tan\left(\frac{146^\circ 50.0'}{2}\right) = 671.600 \text{ ft}$$



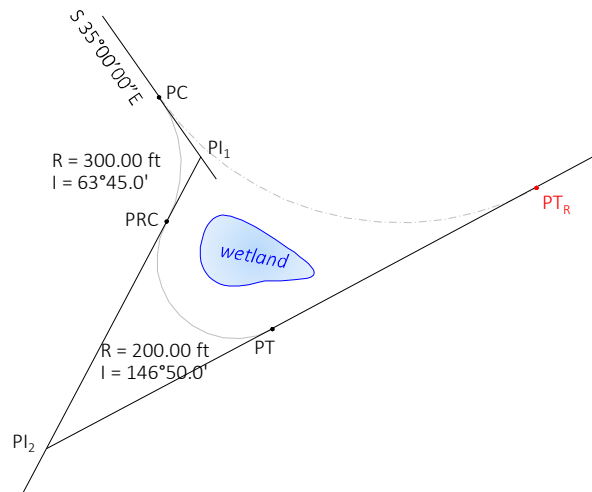
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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

Starting with 2000.00 ft N and 1000.00 ft E for the PC and using the azimuths and distances from the previous slide, the other coordinates are:

Point	N (ft)	E (ft)
PC	2000.00	1000.00
PI ₁	1847.186	1107.002
PRC	1683.630	1017.272
PI ₂	1094.821	694.24
PT	1410.980	1286.769





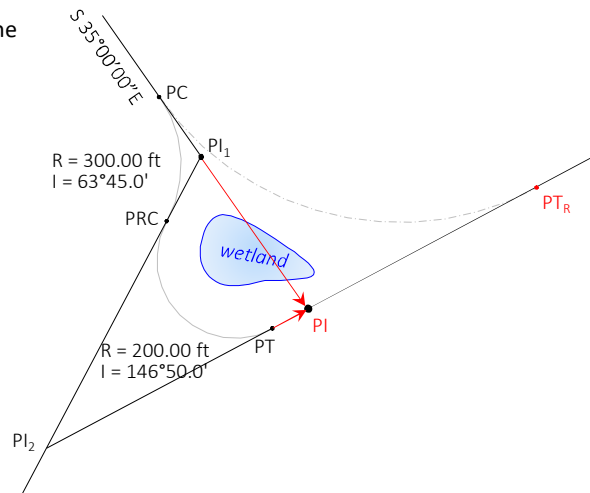
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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

The PI of the new curve is located as the intersection of the incoming and outgoing tangents. Its coordinates are computed with a COGO direction-direction intersection.

Point	N (ft)	E (ft)
PI	1459.795	1378.256



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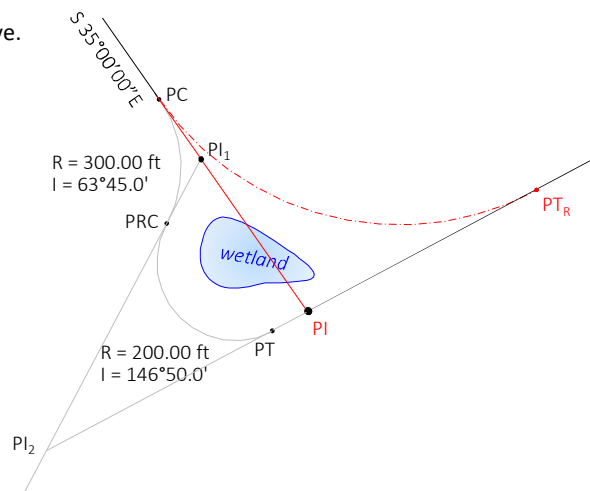
Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

The distance PC-PI is the tangent distance of the new curve. Inverse between coordinates to get the length: 659.468 ft

$$T_1 = R \times \tan\left(\frac{I}{2}\right)$$

$$\Rightarrow R = \frac{T}{\tan\left(\frac{I}{2}\right)} = \frac{659.48 \text{ ft}}{\tan\left(\frac{83^\circ 05.0'}{2}\right)} = 744.315 \text{ ft}$$

Answer: c. 744.3 ft

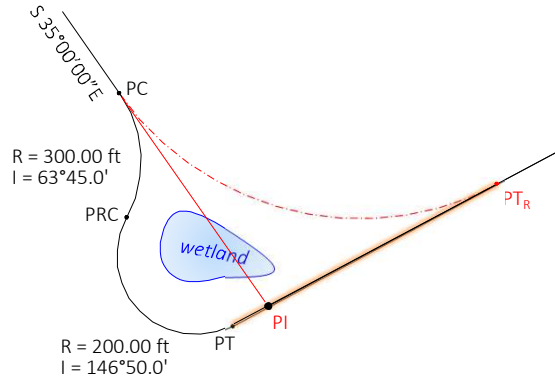


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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

- Part E.2** What is the distance, to the nearest 0.1 ft, along the tangent from the old PT to the new PT_R ?
- a. 536.4 ft
 - b. 601.2 ft
 - c. 688.0 ft
 - d. 763.2 ft



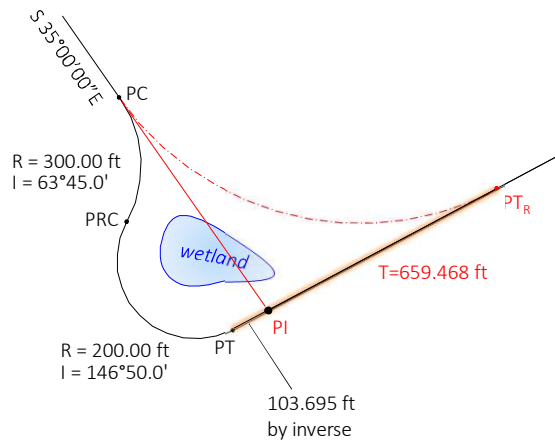
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Problem E. A reverse curve along a road center line is to be replaced by a single tangent horizontal curve. The new curve must begin at the same PC but end at a new PT_R on the same outgoing tangent. The data shown is for the existing reverse curve situation.

- Part E.2** What is the distance, to the nearest 0.1 ft, along the tangent from the old PT to the new PT_R ?
- a. 536.4 ft
 - b. 601.2 ft
 - c. 688.0 ft
 - d. **763.2 ft**

Dist = 103.695 ft + 659.468 ft = 763.163 ft
 Answer: d. 763.2 ft

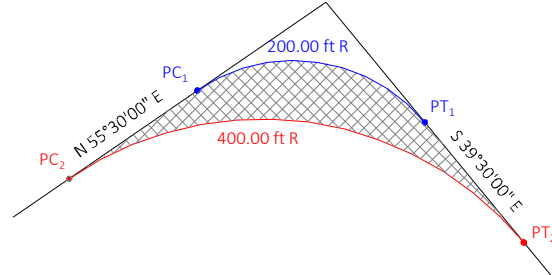


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Problem F. To the nearest 10 square feet, what is the area between the two tangent circular arcs and tangent lines.

- a. 20,950 sf
- b. 29,770 sf
- c. 118,680 sf
- d. 139,630 sf

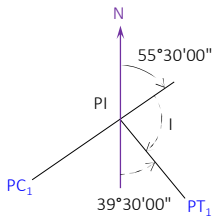


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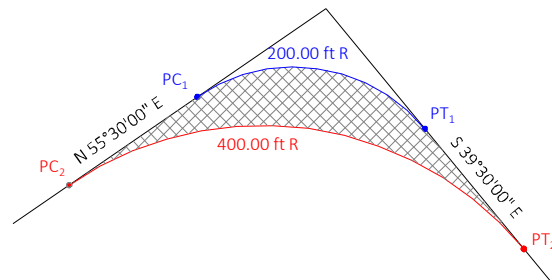
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Problem F. To the nearest 10 square feet, what is the area between the two tangent circular arcs and tangent lines.

Preliminary comps:
determine I angle



$$I = 180^\circ 00' 00'' - (55^\circ 30' 00'' + 39^\circ 30' 00'') \\ = 85^\circ 00' 00''$$



tangent distances

$$T_1 = 200.00 \text{ ft} \times \tan\left(\frac{85^\circ 00' 00''}{2}\right) = 183.266 \text{ ft}$$

$$T_2 = 400.00 \text{ ft} \times \tan\left(\frac{85^\circ 00' 00''}{2}\right) = 366.532 \text{ ft}$$

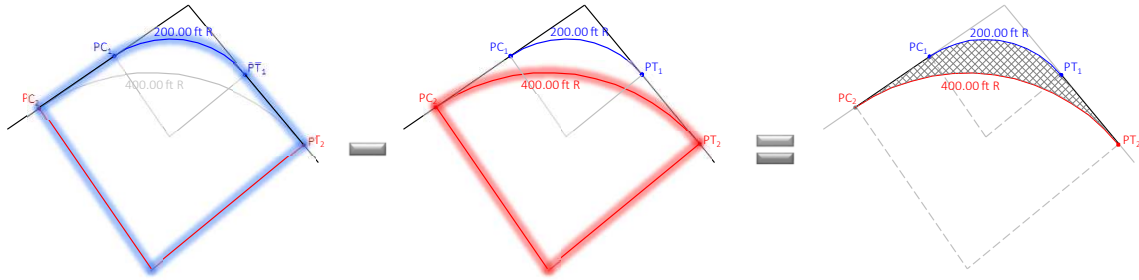
$$T_2 - T_1 = 366.532 \text{ ft} - 183.266 \text{ ft} = 183.266 \text{ ft}$$

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Math Problem Examples

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Problem F. To the nearest 10 square feet, what is the area between the two tangent circular arcs and tangent lines.
Solution Logic

1. Compute the area bounded by the 200.00 ft arc, tangents, and 400.00 ft radii.
2. Subtract the sector area of the 400'00 ft R arc.

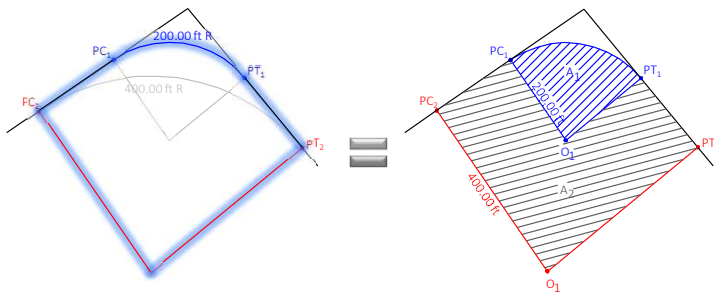


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Math Problem Examples

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Problem F. To the nearest 10 square feet, what is the area between the two tangent circular arcs and tangent lines.
Solution Logic

1. Area bounded by the 200.00 ft arc, tangents, and 400.00 ft radii.
This area is the sum of A_1 and A_2
 A_1 is the 200.00 ft R sector area
 A_2 is bounded by straight lines – compute by coordinates.



$$A_1 = \frac{85^{\circ}00'00'' \times \pi \times (200.00 \text{ ft})^2}{360^{\circ}}$$

$$= 29,670.6 \text{ sf}$$

$$A_2 = 109,959.6 \text{ sf}$$

(from coord comps; not shown)

$$A_1 + A_2 = 139,630.2 \text{ sf}$$



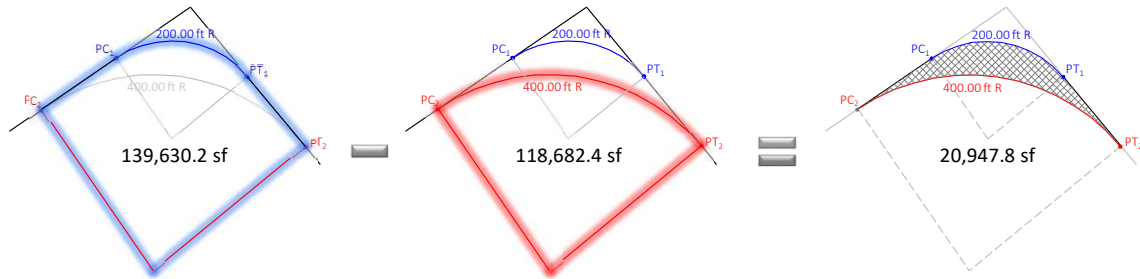
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Problem F. To the nearest 10 square feet, what is the area between the two tangent circular arcs and tangent lines.
Solution Logic

1. Compute the area bounded by the 200.00 ft arc, tangents, and 400.00 ft radii.
2. Subtract the sector area of the 400'00 ft R arc.

$$A_3 = \frac{85^\circ 00' 00'' \times \pi \times (400.00 \text{ ft})^2}{360^\circ} = 118,682.4 \text{ sf}$$



Answer: a. 20,950 sf